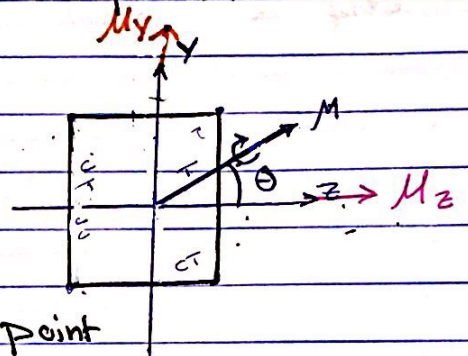


6.5: Unsymmetric Bending

► When M is not applied on a principal axis we have to find its components on the principal axes.

$$M_y = M \sin \theta$$

$$M_z = M \cos \theta$$



► and to find stress at any point or corner :-

$$\sigma = \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

Where: I_z : moment about z axis (of the cross section ex: square, triangle, ...)

I_y : ~ ~ ~ y ~ ~ ~ (~ ~ ~)

y : \perp Distance between point and ~~z~~ axis of moment on y
 z : \perp ~ ~ ~ ~ ~ ~~z~~ axis of moment on z

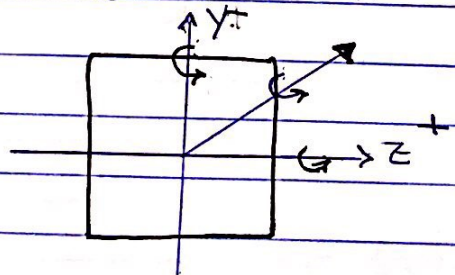
► Sign Convention:

Take each component of the moment

→ M_z :

The direction of M_z (right hand thumb) is at z^+

and The direction of M_y (~ ~ ~) ~ ~ ~ y^+



→ And for the stress: Tension +
compression -

• M_z :-

If M_z is compressing a point then it's neg
" " " tensioning " " " pos

▶ Purple region
is Tension
so M_z is + at
each point



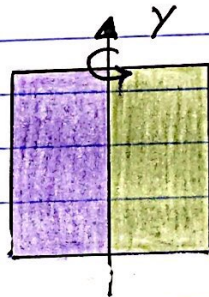
▶ Green region
is Compression
so M_z is - at each point

• M_y :-

Same idea

▶ Purple region
is Tension

▶ Green region
is Compression



→ Orientation of the Neutral axis:-

$$\tan \alpha = \frac{I_z}{I_y} \tan \beta$$

G: + From z^+
to y^+

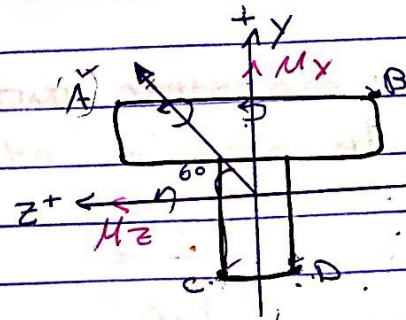
Ex: 6-108

$$(\sigma_{allow})_t = 4 \times 10^6 \text{ Pa}$$

$$(\sigma_{allow})_c = 6 \times 10^6 \text{ Pa}$$

$$M_y = M \sin 60$$

$$M_z = M \cos 60$$



Find \bar{y} ($\bar{y} = 0.0875$)

Find I_z .

$$I_z = \left(\frac{1}{12}\right)(0.2)(0.05)^3 + (0.05)(0.2) \left(\frac{1}{2}\right)(0.05)(0.2)^2 + \left(\frac{1}{12}\right)(0.05)(0.2)^3 + (0.05)(0.2)(0.15 - 0.0875)^2 = 0.1135 \times 10^{-3}$$

σ is Tension at C
Totally

$$\sigma = \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$4 \times 10^6 = \left(\frac{M \cos 60}{0.1135 \times 10^{-3}}\right) (0.025)$$

$$+ \left(\frac{M \sin 60}{35.417 \times 10^{-6}}\right) (0.025)$$

$$I_y = \left(\frac{1}{12}\right)(0.05)(0.2)^3 + \left(\frac{1}{12}\right)(0.2)(0.05)^3 = 35.417 \times 10^{-6}$$

$$M = 2119.71 \text{ N}\cdot\text{m}$$

σ is Totally Compression at B

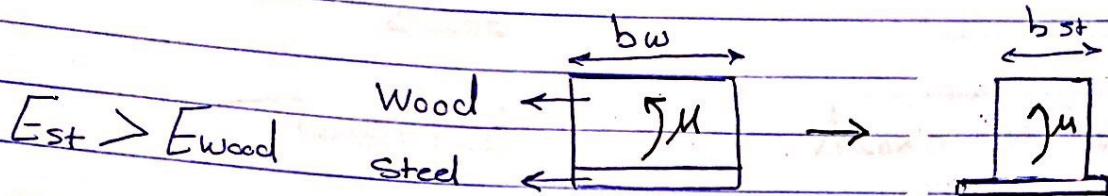
$$\sigma = -\frac{M_z}{I_z} y + -\frac{M_y}{I_y} z$$

$$M = 3.01 \text{ kN}$$

→ Take the less value.

6.6 Composite Beams:

→ If you have a piece of two materials you can transform it to a piece of one material.



$$b_{st} = n b_w \rightarrow \text{where } n = \frac{E_w}{E_{st}} \frac{\text{mat (1)}}{\text{mat (2)}}$$

- after finding new b and n find \bar{y} (Neutral axis) and I_{NA}
- and you can use $\sigma = \frac{M c}{I}$ to find stress

If you want to find σ at a point but in wood use

$$\sigma_w = n \sigma_{st}$$

old mat (1) new mat (2)

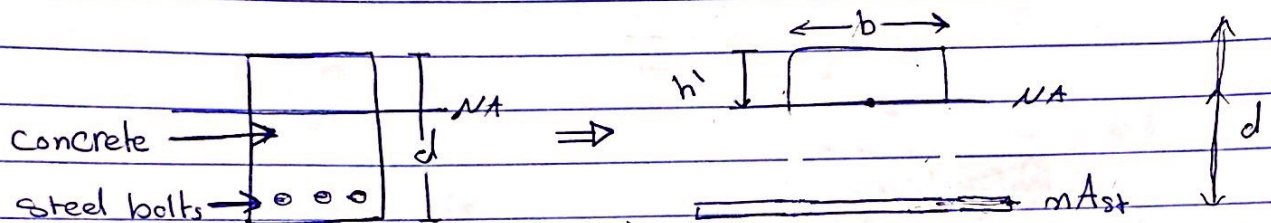
G.F: Reinforced Concrete Beams

$$n = \frac{E_{st}}{E_{conc}}$$

Transformed Area : $A_{con} = n A_{st}$

Equation to Find h' : (Distance From top to N.A.)

$$\rightarrow \frac{b}{2} h'^2 + n A_{st} h' - n A_{st} d = 0$$



Sketch of stress: Comp ←

← Tension

$$\Sigma \bar{y} A = 0$$

Ch7. Transverse Shear

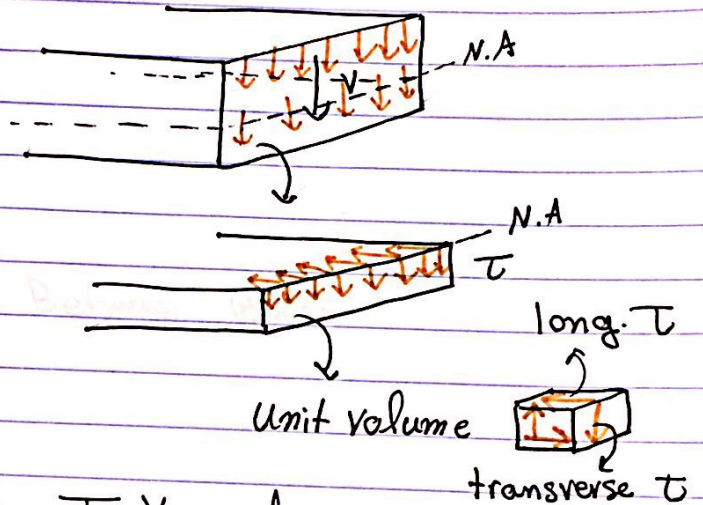
F.1+2: Shear in straight members and shear formula.

Shear $\left\{ \begin{array}{l} \rightarrow \text{longitudinal shear} \\ \rightarrow \text{transverse shear} \end{array} \right.$

• Shear stress at the top = 0

$$\bar{\tau} = \frac{VQ}{It}$$

and τ_{max} is at the N.A.



► Difference between $\bar{\tau} = \frac{V}{A}$ and Shear formula $\bar{\tau} = \frac{VQ}{It}$

$\bar{\tau} = \frac{V}{A}$ calculates Avg shear stress in the cross section

$\bar{\tau} = \frac{VQ}{It}$ ~ shear stress in a specific point or Area

Now: what's t and Q :-

Q : Moment of Area above or under point

t : width of cross sectional area at where τ is measured.

7.3: Shear Flow in Built up Members.

$$\text{Shear Flow } q = \frac{VQ}{I}$$

q : is the loading measured as a force per unit length

You can relate Shear Flow and Shear Stress

$$\tau = \frac{q}{t}$$

→ Now:

$$F = \frac{s \times q}{n}$$

spacing Between Bolts

Strength of each nail

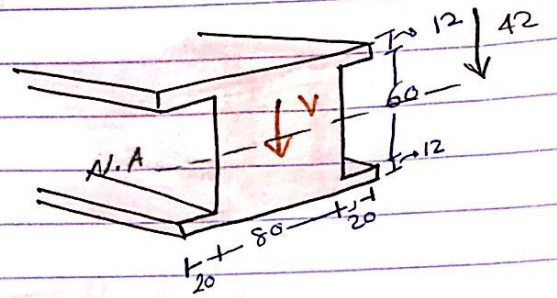
n : number of nails crossing the Area Boundary (see page 382 in Book)

► to find shear stress in each Bolt

$$\tau = \frac{F}{A} = \frac{s \times q}{n \times A}$$

Examples solved on Ch 7:

7.14 $V = ?$
max



$$\tau_{all} = 40 \text{ MPa}$$

$$\tau_{all} = \frac{VQ}{It}$$

→ Q_{max} is at N.A

• $t = 80 \text{ mm}$

$$I = 2\left(\frac{1}{12}(120)(12)^3 + (12)(120)(36)^2\right) + \left(\frac{1}{12}\right)(80)(60)^3$$

$$= 5207040 \text{ mm}^4 = 5207.040 \times 10^{-12} \text{ m}^4$$
$$= 5.20704 \times 10^{-6} \text{ m}^4$$

$$Q = \sum \bar{y}A = (0.036)(0.012)(0.12) + \left(\frac{0.03}{2}\right)(0.08)(0.03)$$
$$= 8.784 \times 10^{-5}$$

$$40 \mu = \frac{V(8.784 \times 10^{-5})}{(5.20704 \times 10^{-6})(0.08)}$$

$$V = 189691.8 \text{ N}$$

7.41

$$P = 12 \text{ kN}$$

$$S = ?? \quad \text{if } F_{\text{max}} = 1.5 \text{ kN}$$

$$F = \frac{q \times S}{n}$$

$$S = \frac{\bar{n} F}{q}$$

$$q_{\text{max}} = \frac{V_{\text{max}}}{I} =$$

Q_{max} (Around N.A)

$$Q = (12.5)(0.025)(0.1)$$

$$Q = 2.8125 \times 10^{-1}$$

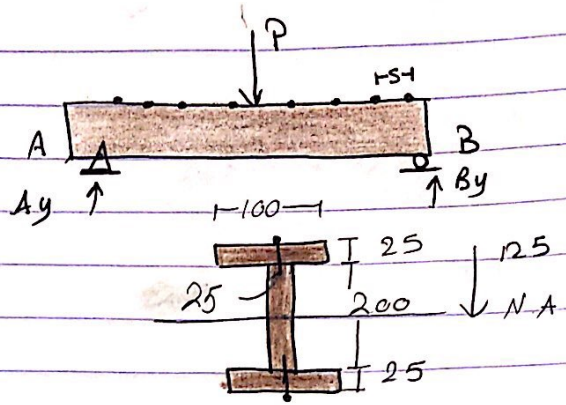
$$\text{So } q_{\text{max}} = \frac{(6 \text{ k})(2.8125)(10^{-1})}{(8.0208)(10^{-5})}$$

$$= 21039.048 \text{ N/m}$$

$$S = \frac{(1)(1.5 \times 10^3)}{(21039.048)}$$

$$S = 0.071296 \text{ m}$$

$$= 71.296 \text{ mm}$$



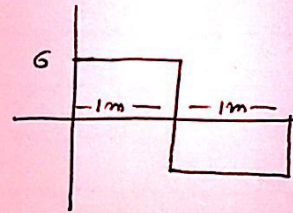
To find V_{max} :

• Reactions :

$$\sum M_A = 0 : -P(1) + (B_y)(2) = 0$$

$$B_y = 6 \text{ kN} = A_y$$

• So $V_{\text{max}} = 6 \text{ kN}$



$$I = 2 \left[\frac{1}{12} (0.1)(0.025)^3 + \frac{(0.1)(0.025)}{(0.1125)^2} \right]$$

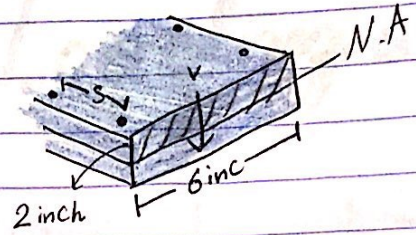
$$+ \frac{1}{12} (0.025)(0.200)^3$$

$$I = 8.0208 \times 10^{-5}$$

F-33

$$S = 6 \text{ inch}$$

$$V = 600 \text{ lb}$$



$$F = \frac{q \times S}{n}$$

$$F = \frac{q \times 6}{2 \sim 2 \text{ rows}}$$

$$q = \frac{VQ}{L} = \frac{600 \left(\frac{11}{12} (2) (6) \right)}{(12) (6) (4)^3 \text{ in}^4} = 225 \text{ lb/in}$$

$$F = \frac{(225)(6)}{2} = 675 \text{ lb}$$

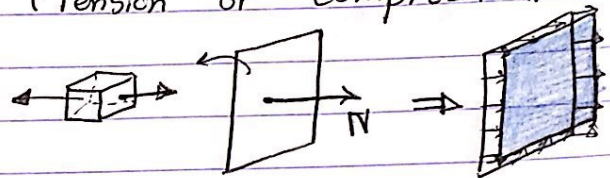
Ch 8. Combined Loadings.

→ 8.2: State of stress caused by combined Loadings.

In this section you will have to find the combined state of stress at a point
So Remember:

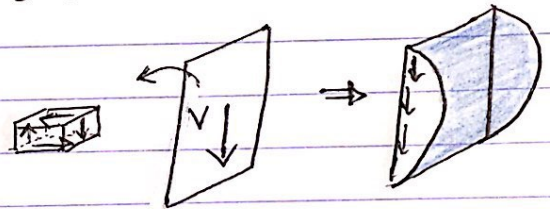
Normal Force → It Does a normal stress (Tension or compression)

$$\sigma = \frac{N}{A}$$



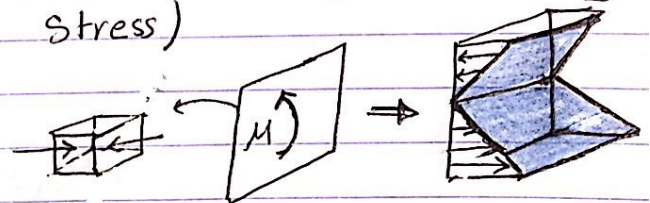
Shear Force → It Does a shear stress

$$\tau = \frac{VQ}{It}$$



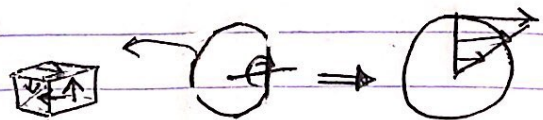
Bending Moment → It Does a stress (Bending stress)

$$\sigma = \frac{Mc}{I}$$



Torsional Moment → It Does a shear stress

$$\tau = \frac{Tc}{J}$$

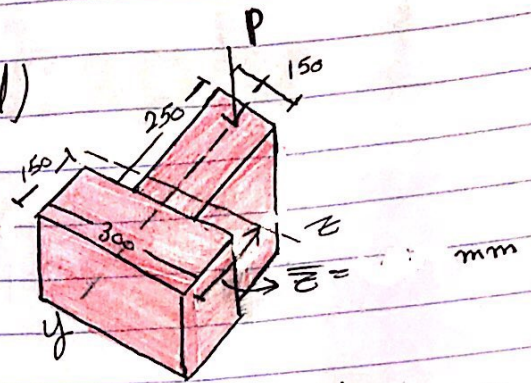


8-27

$$\sigma_{all} = 6 \text{ MPa (for wood)}$$

$$P_{max} = ?$$

location of \bar{z} :

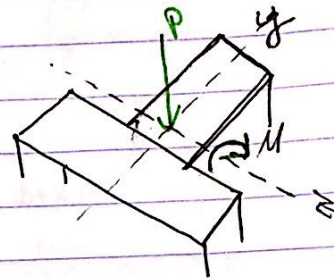


$$\bar{z} = \frac{(0.15)(0.3)(0.15) + (0.3)(0.3)(0.15)}{(0.3)(0.15) + (0.15)(0.3)} = 1.044 \text{ mm}$$

Moving P:

$$M = (P) \left(\frac{40187.5}{1000} \right)$$

$$M = 0.2125 P$$



There are two stresses

$$\sigma_{all} = \sigma_p + \sigma_M$$

$$= \frac{P}{A} + \frac{(M)c}{I}$$

$$= \frac{P}{(0.3)(0.15) + (0.15)(0.3)} + \frac{(0.2125 P)(0.2625)}{I_z}$$

Finding $I_z = 1.5609 \times 10^{-3}$

$$6 \times 10^6 = 11.11 P + 35.736 P$$

$$P = 128 \text{ kN}$$