



**BIRZEIT UNIVERSITY**  
Faculty of Engineering and Technology

Department of Mechanical and Mechatronics Engineering

ENME 435: Machine Design 1– First Exam

Date of Examination: 28.10.2019

Instructor: Dr. Rashad Mustafa

Student Name: \_\_\_\_\_

Time duration: 90 minutes

Total Marks: 100%

The pipe assembly is fixed at **A**. The rod has a diameter of 25 mm and made of A-36 steel. Please answer the following questions:

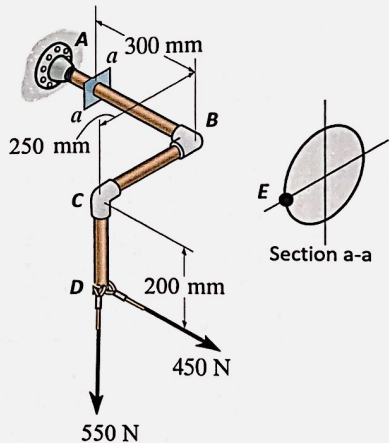
**Question 1:** Please, determine the state of stress at point **E** on the cross section of the rod at section a-a. (25P)

**Question 2:** Please, determine and draw the principal stress and maximum in-plane shear stress at Point **E** on the cross section of the pipe at section a-a. (15P)

**Question 3:** Please determine the total elastic strain energy that can be stored in the rod **A-B**. The distance between **A-B** is 400 mm. Neglect the shearing strain energy. (20P)

**Question 4:** Please, determine the factor of safety for the above given material for all applicable theory. (20P)

**Question 5:** What diameter should be used for the shaft (**A-B**) in order to get a factor of safety **10**, if the rod **A-B** is to be made of Cast Iron Alloys –ASTM A-197. Consider please for this calculation only an axial tension force of **55 kN** applied on the A-B member. (20P)



$Q_f ::$

$d = 25 \text{ mm}$

$\sum F_y = 0$

$V_y - 550 = 0$

$V_y = 550 \text{ N} \checkmark$

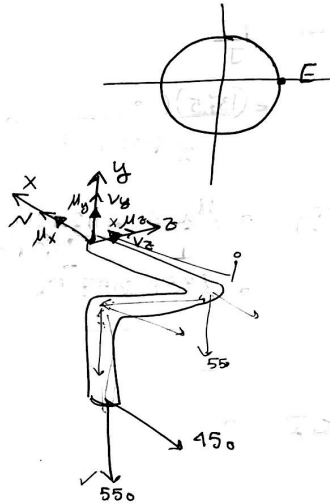
$\sum F_z = 0$

$V_z = 0$

$\sum F_x = 0$

$N - 450 = 0$

$N = 450 \text{ N} \checkmark$



Moments

$\sum M_x = 0 : M_x - (550)(0.25) = 0$

$M_x = 137.5 \text{ N.m} \checkmark$

$\sum M_y = 0 : (450)(0.25) + M_y = 0$

$M_y = -112.5 \text{ N.m}$

$\sum M_z = 0 : M_z + (0.3)(550) - (450)(0.2) = 0$

$M_z = -75 \text{ N.m}$

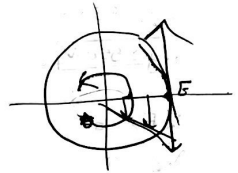
Stresses we have at E:  $\tau_{xy}, \tau_x, \sigma_N, \sigma_y, \sigma_z$

$\sigma_N = \frac{N}{A} = \frac{450}{\frac{\pi}{4}(0.025)^2} = 917197.45 \text{ Pa} \text{ (Tension)}$

$$\tau_x = \frac{Tc}{J}$$

$$= \frac{(137.5)(0.0125)}{\frac{\pi}{2}(0.0125)^4} = 44840764.33$$

$$\sigma_y = \frac{M_x z}{I} = \frac{(112.5)(0.0125)}{1.9165 \times 10^{-8}} = 73375945.7 \text{ Pa}$$



$$I = \frac{\pi}{4}(R)^4$$

$$= \frac{\pi}{4}(0.0125)^4$$

$$= 1.9165 \times 10^{-8}$$

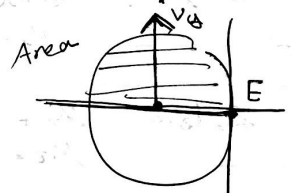
$$\sigma_z = 0$$

$$\tau_{xy} = \frac{VQ}{Ib}$$

$$= \frac{(550)(QA)}{Ib}$$

$$= \frac{(550) \left( \frac{4 \times 0.0125}{3\pi} \right) \left( \frac{1}{2} \pi (0.0125)^2 \right)}{1.9165 \times 10^{-8} \times 0.025}$$

$$= 1494695.191 = 1.495 \text{ MPa}$$



• So we have three stresses

$$\sigma_y, \tau_x, \sigma_z$$

$$\sigma = +73.4 \text{ MPa} + 0.9179 \text{ MPa}$$

$$\sigma = 74.29 \text{ MPa} \text{ Tension}$$

$$\tau_x = 44840764.33 \Rightarrow \tau = 44.84 \text{ MPa}$$

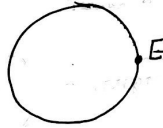
$$\tau_{tot} = 44.84 + 1.495 = \cancel{46.335} \text{ MPa} = 46.335 \text{ MPa} \downarrow$$

Q2:-

$$\frac{F_{xy}}{L_{xy}} = 46.335 \downarrow \text{ (neg)}$$

$$\sigma_x = 74.29 \text{ MPa}$$

$$\sigma_y = 0$$



Now:-

Max the principal stresses:-

$$\sigma_1, \sigma_2$$

$$\begin{aligned} \sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{74.29}{2} \pm \sqrt{\left(\frac{74.29}{2}\right)^2 + (46.335)^2} \end{aligned}$$

$$\sigma_{1,2} = 37.145 \pm 59.386$$

$$\sigma_1 = 96.53$$

$$\sigma_2 = -22.241$$

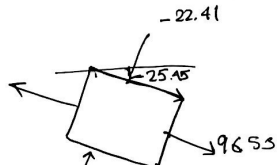
Now we find  $\theta_p$ :-

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{-46.335}{\frac{74.29}{2}}$$

$$\frac{2\theta_p}{\theta_p} = \frac{-50.9}{-25.15}$$

$$\sigma_x' = \frac{74.29}{2} + \frac{74.29}{2} \cos(-50.9) + (-46.335) \sin(-50.9)$$

$$\sigma_x' = 96.54 \rightarrow \text{so element}$$



Max Shear:-

$$\tau_{xy\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= 59.39 \text{ MPa}$$

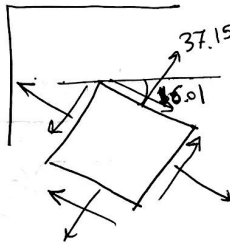
$$\sigma_{avg} = 37.15 \text{ MPa}$$

We need  $\theta_s$ :-

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

$$2\theta_s = -32.03$$

$$\theta_s = -16.01$$



To find  $U_{AB}$ :-

$$U_{AB} = U_N + U_{M_y}$$

$$= \frac{N^2 L}{2AE} + \int_0^{400} \frac{M^2}{2EI} dx$$

$$= \frac{(450)^2 (0.4)}{(2)(0.025)^2 \frac{\pi}{4} (200 \times 10^9)}$$

$$+ \int_0^{400} \frac{(137.5)^2 dx + (112.5)^2 dx + (90 - 550x)^2 dx}{2(200 \times 10^9) \times \frac{\pi}{4} (0.025)^4}$$

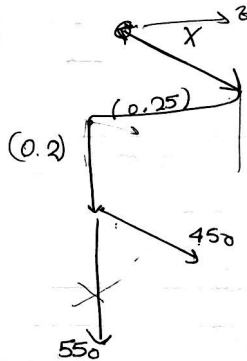
$$= 4.127 \times 10^{-1} J + \int$$

$$M_x = (0.25)(550) = 137.5$$

$$\frac{M_z + (550)(x) - (450)(0.2) = 0}{M_z = 90 - 550x}$$

$$M_y + (450)(0.25) = 0$$

$$M_y = -112.5$$



$$\frac{1}{2} \frac{I}{GJ}$$

1.3

$$\frac{(0.1)(137.5)^2}{(2)75 \times 10^9 \times \frac{\pi}{4} (0.0125)^4}$$

$$= 4.127 \times 10^{-1} + 0.4 \left( \frac{137.5^2 + (112.5)^2}{7666} \right) + \int_0^{0.1} \frac{(90 - 49500x)^2}{7666} dx$$

$$= 4.127 \times 10^{-1} + \frac{1646.9}{1.00} + \frac{(8100)(0.1) - (49500) \frac{x^2}{2}}{7666} \Big|_0^{0.1}$$

$$+ \frac{302500 \frac{x^3}{3}}{7666} \Big|_0^{0.1}$$

$$= 2.717 J$$