

# Chapter 15:

15.1: Principle of linear mom & impulse

- This method is used when you have Forces, velocities, time.

→ Forces should be external

Ex: 15-11

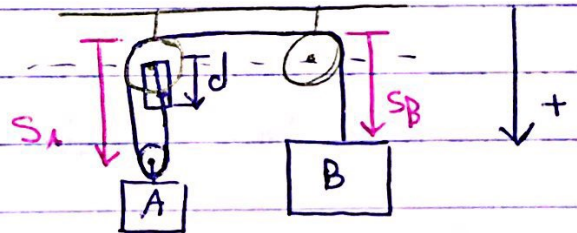
As you see in the Question it asked for the velocity after 2s (Time is Given)

$$W_A = 10 \text{ Ib}$$

$$W_B = 50 \text{ Ib}$$

$$L = S_A + S_{A-d} + S_B$$

$$L = 2S_A + S_B$$

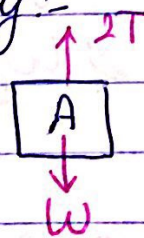


$$\text{So } 0 = 2V_A + V_B \rightarrow \text{eq 1}$$

Now, you can use principle of linear momentum for Each Body:-

$$\text{For A: } \Sigma F = W - 2T$$

$$V_i = 0 \quad (\text{since it started from rest})$$



$$m v_{A1} = 0$$

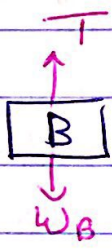
$$m v_{A2} = ?$$

So:

$$m v_{A1} + \int_0^2 \Sigma F dt = m v_{A2}$$

$$\int_0^2 W_A - 2T dt = m_A v_{A2} \rightarrow \text{eq 2}$$

for B:

$$\text{eq 3} \leftarrow \int_0^2 W_B - T dt = m_B v_{B2}$$


You solve 2 with 3 using 1

$$2(W_A - 2T) = m_A v_{A2} \quad \text{From eq 1}$$

$$2(W_B - T) = m_B (-2v_{A2}) \quad \times 2$$

$$2W_A - 4T = m_A v_{A2}$$

$$4W_B - 4T = -4m_B v_{A2}$$

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$$2W_A - 4W_B = m_A v_{A2} + 4m_B v_{A2}$$

$$2 \times 10 - 4 \times 50 = \left[ \frac{10}{32.2} + (4) \times \frac{50}{32.2} \right] V_{A2}$$

$$-180 = 6.52 - V_{A2}$$

$$V_{A2} = -27.6 = 27.6 \uparrow$$

$$\text{So } V_{B2} = 55.2 \downarrow$$

15.3: Conservation of linear momentum + Impact

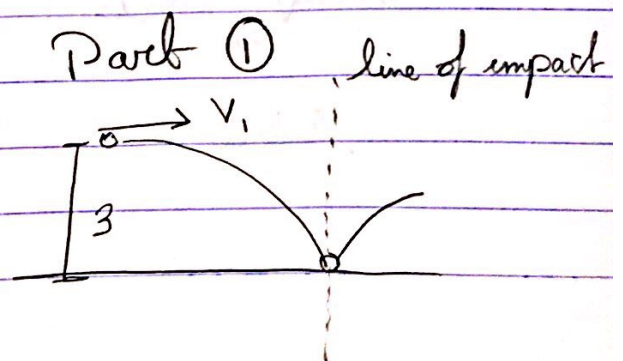
$$mV_1 = mV_2$$

Happens when  $\Sigma F$  external is zero.

- Collision: You have to define l.o.f impact
- Movement with NO friction
- projectiles

Ex 15-68, projectile + collision

$e = 0.8$   
 ↑ This is used for  $V_y$  (line of impact)



So:-

$V_g$ , Velocity of Ground

$$e = \frac{V'_{by} - V_{gy}}{V_{gy} - V_{by}}$$

$-0.8 V_{by} = V'_{by}$  ← هذا النقص  
قبل القيام بحركة  
في اتجاهها



$$V_{by}^2 = V_{by0}^2 + 2a\Delta y$$

$$V_{by}^2 = (2)(32.2)(3)$$

$$\boxed{V_{by} = 13.9} \downarrow$$

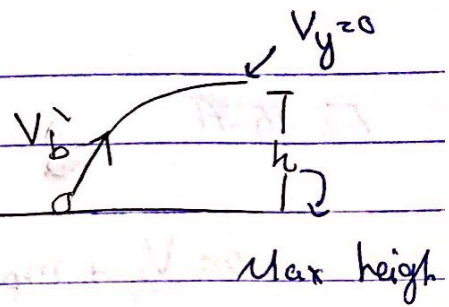
$$So: -(0.8)(-13.9) = V'_{by}$$

$$\boxed{V'_{by} = 11.1} \uparrow$$

$V_{bx}$  is constant = 8 ft/s

Now Part 2:

$$V_{by}^2 = \cancel{V_y^2} + 2a \Delta y$$



$$123.648 = -2 \times 32.2 \times h$$

$$h = 1.92 \text{ ft}$$

Note: collision has 3 cases

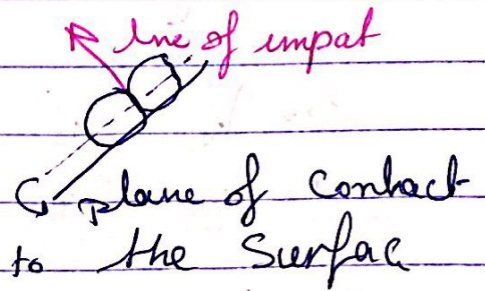
$e=0$  : particles stick together after collision

$e=1$  : No loss in Energy

$0 < e < 1$  : Normal collision (loses in Energy)

- $e$  is always used for velocities related to line of impact and the other velocities are conserved.

Ex: usually line of impact is the **Normal** to the surface



# Central impact

Ex: 15-ff

$$m_A V_{A1} + m_B V_{B1} = m_A V_{A2} + m_B V_{B2}$$

Since masses are equal:-

$$5 = V_{A2} + V_{B2} \quad \text{--- (1) } \leftarrow +$$

Using  $e$ ,  $e = \frac{V_{A2} - V_{B2}}{V_{B1} - V_{A1}}$

$$0.8 = \frac{V_{A2} - V_{B2}}{0 - 5}$$

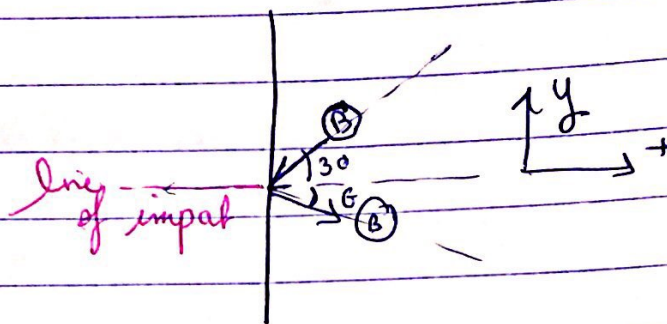
$$-4 = V_{A2} - V_{B2} \quad \text{--- (2)}$$

Solving,

$$\begin{aligned} 5 &= V_{A2} + V_{B2} \\ -4 &= V_{A2} - V_{B2} \end{aligned} \quad +$$

$$\boxed{V_{A2} = 0.5} \rightarrow V_{B2} = 4.5 \quad \leftarrow +$$

Oblique impact



$$e = \frac{V_B'}{V_B \cos \theta}$$

$$0 = \underset{\substack{\uparrow \\ \text{wall}}}{V_{B2}} \cos 30^\circ$$

$$0.6 (V_{B2} \cos 30^\circ) = V_B' \cos \theta$$

$$2.34 = V_B' \cos \theta \rightarrow (1)$$

$$V_{B2} \sin 30^\circ = V_B' \sin \theta$$

$$\textcircled{2} \quad 2.25 = V_B' \sin \theta \rightarrow (2)$$

Divide  $\textcircled{2}$  by  $\textcircled{1}$

$$\theta = 43.88$$

$$V_B' = 3.24$$

## \* Angular Momentum

-  $H = \vec{r} \times m\vec{V}$  ← used when asked to find Angular Momentum

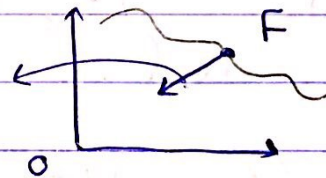
- Also,  $\Sigma M_o = \dot{H}_o$   
↑  
مشتقة الزخم الزاوي بالوقت

- And so:  $H_{o1} + \int_{t_1}^{t_2} M_o dt = H_{o2}$

↑ used when you are given forces and moments

- Conservation:  $(H_{o1}) = (H_{o2})$

• This force does no moment...



• parallel forces to axis does no moment around the axis.



Ex:- 15-102:

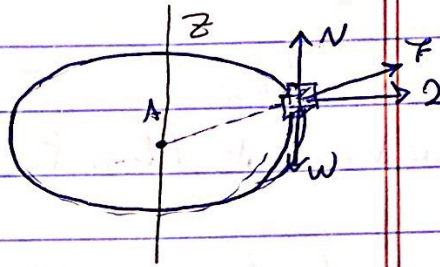
In this Question There are 4 forces

$$F_1 = 2Ib$$

$$F_2 = 7Ib$$

$$F_3 = W$$

$$F_4 = N$$



Take H around z axis:-

- N, W Does no Moment around it
- 2Ib can't too
- The only force affecting is  $7 \cos 30$

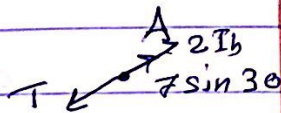
Now:  $H_2^0 + \int_{t_1}^{t_2} \Sigma M dt = H_2$

①  $\int_0^t (7 \cos 30) (1) dt = r \times m V_1$  *You have to find it*

→ To find  $V_1$ , use Newton's law

Use  $\Sigma F_m$ :-

$30 \rightarrow T - 2 - 7 \sin 30 = \frac{(10)}{32.2} \frac{V^2}{4}$



$V = 17.76$  put in ① :  $t = 0.991$  s