Chapter 16

- 16.1 + 16.2 + 16.3 (Rules are in the Summary)

$\vec{V} = \omega \times r$ is used when there is a rotation around a fixed axis

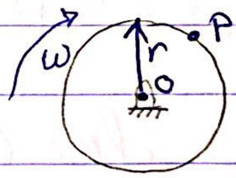
ω: is the Angular velocity of the Rigid body

r: distance between axis and point

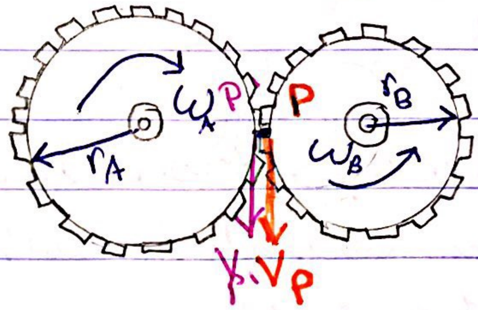
- Types of Questions in these Sections

1- Circular body Rotating around point

$v_p = r\omega$ (scalar) Multiplication



2- Gears:



$\boxed{\omega_A r_A = r_B \omega_B}$

→ always

$V_{p'} = V_p$

$a_{p't} = a_{pt}$ (only tangential)

$\boxed{\alpha_A r_A = r_B \alpha_B}$

→ always

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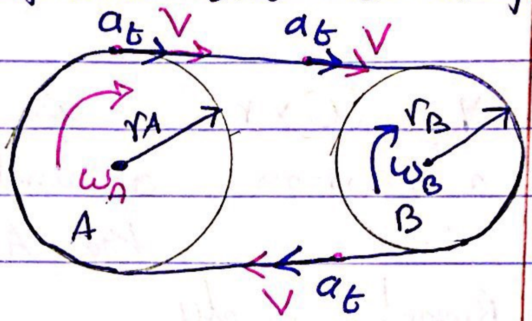
3- Belts :-

- Belts translate Motion so V, a in a belt is equal in each point.

$V = r_A \omega_A = r_B \omega_B$

$r_A \theta_A = r_B \theta_B$



Examples:-

16-29:-

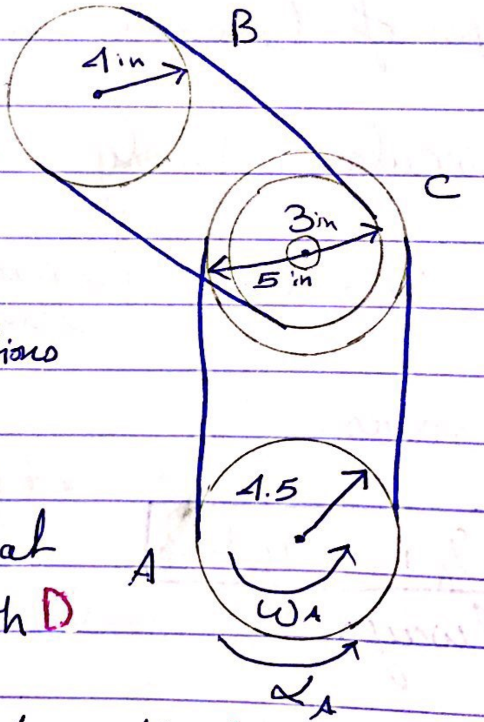
$\omega_{A_0} = 5$ rad/s

$\alpha_A = 2$ rad/s²

- ω_B after 6 revolutions of B

- We can see that B is connected with D with a Belt

And C is connected with A with a Belt



First : $\theta_B = 6$ revolutions

$\theta_B r_B = \theta_C r_D$

$(6)(4) = \theta_C \times \cancel{5}\, 3$

$\underline{\theta_D = \cancel{8}\, 8} = \theta_C$ since D is put on C

* We need ω_2 of C to find ω_2 of

$\omega_{C_1} r_C = \omega_{A_1} r_A$

for C:- $\omega_{C_2}^2 = \omega_{C_1}^2 + 2\alpha_c (\Delta\theta)$ $\quad 4.8 \times 2\pi$

$\alpha_c r_C = \alpha_A r_A$

$\omega_{C_2}^2 = (4.5)^2 + 2 \times 1.8 \times \cancel{8}\, 8 \times 2 \times 3.1$

$\omega_{C_2} = 14.\cancel{1}8$

Now $\omega_{C_2} r_D = r_B \omega_{B_2}$

$\omega_{B_2} = 10.6$ rad/s

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$16-13$:- $\alpha_A = 4t^3$ $\quad (\omega_A)_0 = 20$ rad/s

$\omega_B = ??$ at $\underline{t = 2s}$

$\omega_B r_B = \omega_A r_A$

$\omega_B = \omega_A \; 0.33$

$$\int_{20}^{\omega} d\omega_A = \int_0^2 \alpha \, dt$$

$$\omega - 20 = \frac{4t^4}{4}\Big]^2$$

$\omega_{A_2} = 36$

$\omega_B = 36 \times 0.33 \doteq 12$

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• Relative Motion Analysis :-

Velocity and acceleration

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$\vec{V}_B = \vec{V}_A + \overbrace{\omega \times r_{B/A}}$

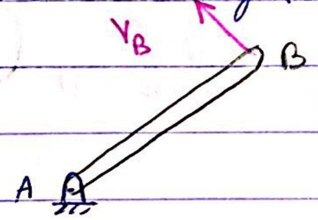
$\vec{a}_B = \vec{a}_A + \underbrace{\alpha \times \vec{r}_{B/A}}_{(a_{B/A})_t} - \underbrace{\omega^2 r_{B/A}}_{(a_{B/A})_n}$

* used when the two point A, B are on the same Rigid body

• Questions Related to this subject :-

$\boxed{A}$ Collars :-

① one side is fixed



• In this case:

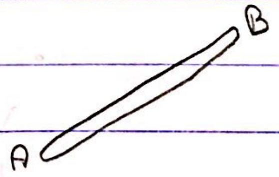
• $\vec{V}_A = 0$

• and V_B is perpendicular to AB

• and you can use

$\vec{V}_B = \cancel{\vec{V}_A}^{0} + \omega \times r_{B/A}$

② two sides are free



• In this case

$\vec{V}_B = \underbrace{\vec{V}_A}_{\neq 0} + \omega \times r_{B/A}$

• And you have to relate each point to a fixed axis or to find IC

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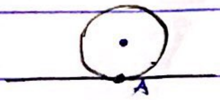
B Wheels :-

on the Ground

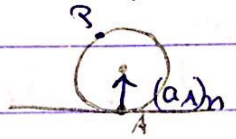
Rotating

slipping



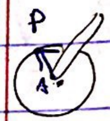
$V_A \neq 0$

Does not slip



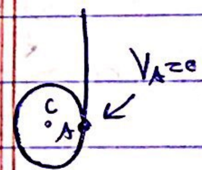
- $V_A = 0$ and $(a_A)_t = 0$ But $(a_A)_n \neq 0$
- Center has v, a in i direction
- P has a tangential velocity and a tangential and normal acceleration

- Subjected to a Collar

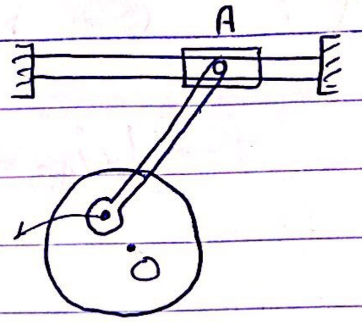


- A is a fixed Axis and $V_A = 0$
- Any point : P has a tangential velocity always perpendicular to $r_{P/A}$
- or Subjected to a Rope

V_c, a_c is up or Down (j)



Example :- 16-110



- We need to find a_A

Now, we can relate A to B

$$\vec{a}_A = \vec{a}_B + \alpha_{AB} \times \vec{r}_{A/B} - \omega_{AB}^2 \times \vec{r}_{A/B} \rightarrow ①$$

We need to find a_B, α_{AB}, ω_{AB}

First $V_0 = 0$

$$\vec{V}_B = \cancel{\vec{V}_0}^{0} + \omega \times \vec{r}_{B/O}$$

$$\vec{V}_B = -(8\hat{k} \times 0.15(-\cos 30\hat{i} + \sin 30\hat{j})$$

$$\vec{V}_B = 1.039\hat{j} + 0.6 \quad \uparrow$$

Now we know that V_A is in $\hat{i}$ Direction

$$V_A\hat{i} = 1.039\hat{j} + 0.6\hat{i} + \omega_{AB}\hat{k}(0.5)(\cos 60\hat{i} + \sin 60\hat{j})$$

$$\vec{V}_A\hat{i} = 1.039\hat{j} + 0.6\hat{i} + 0.25\omega_{AB}\hat{j} + 0.433\,\omega_{AB}\hat{i}$$

$\Sigma\hat{i}$:- $\vec{V}_A = 0.6 + 0.433\,\omega_{AB}$

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$\Sigma \hat{j}$: $0 = 1.039 + 0.25\, \omega_{AB}$

$$\boxed{\omega_{AB} = -4.156\, \hat{k}}$$

- Now we need $\vec{a}_B$:-

$$\vec{a}_B = \cancel{\vec{a}_O}^{\,0} + \alpha \times \vec{r}_{B/O} - \omega^2 \times \vec{r}_{B/O}$$

$$\vec{a}_B = (-16\hat{k})\underset{(0.15)}{(-\cos 30 \hat{i} + \sin 30 \hat{j})} - (64)\,(0.15)(-\cos 30 \hat{i} + \sin 30 \hat{j})$$

$$\vec{a}_B = 2.08\hat{j} + 1.2\,\hat{i} + (0.15)64 \cos 30\, \hat{i} - (0.15)64 \sin 30\, \hat{j}$$

$$\vec{a}_B = -2.72\, \hat{j} + 9.5\, \hat{i}$$

Now we can use eq 1:-

we know it's in i direction

$$\vec{a}_A \hat{i} = 9.5\,\hat{i} - 2.72\hat{j} + (\alpha_{AB}\hat{k}) \times (0.5)(\cos 60 \hat{i} + \sin 60 \hat{j}) - (4.156)^2 \times (0.5)(\cos 60 \hat{i} + \sin 60 \hat{j})$$

$$\vec{a}_A \hat{i} = 9.5\,\hat{i} - 2.72\,\hat{j} + 0.25\,\alpha_{AB}\,\hat{j} + -0.433\,\alpha_{AB}\,\hat{i} - 4.318\,\hat{i} - 7.479\,\hat{j}$$

Σi :- $a_A = 9.5 - 0.433\alpha_{AB} - 4.318$

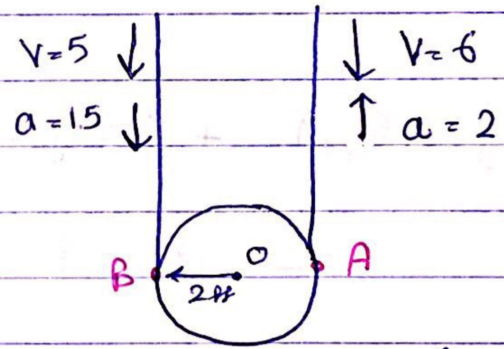
$0 = -2.72 + 0.25\alpha_{AB} - 7.479$

$\alpha_{AB} = 40.796$ $+\circlearrowleft$

So $a_A = -12.48 = 12.48 \leftarrow$

Example :- 16-126.

We need to find ω, α of pipe



The Center has a Velocity and Acceleration up or down

$\vec{V}_A = V_{center} + \omega_A \times r_{A/center}$

$\vec{V}_B = V_{center} + \vec{\omega} \times r_{B/center}$

$\vec{V}_A - \vec{V}_B = (\omega\hat{k} \times +2\hat{i}) - (\omega\hat{k} \times -2\hat{i})$

$-6\hat{j} - (-5\hat{j}) = 2\omega\hat{j} + 2\omega\hat{j}$

$-1 = 4\omega$

$\omega = 0.25 \circlearrowright$

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$$\vec{a}_A = a_{center} + \alpha \times r_{A/center} - \omega^2 r_{A/center}$$
$$\vec{a}_B = a_{center} + \alpha \times r_{B/center} - \omega^2 r_{B/center}$$

$$\vec{a}_A - \vec{a}_B = (\alpha \hat{k} \times 2\hat{i}) - (\alpha \hat{k} \times -2\hat{i})$$
$$- 0.25^2 [2\hat{i} - -2\hat{i}]$$

We have a_A, a_B tangential but
We need the normal so

eq *

$$2\hat{j} + a_{At}\hat{i} + 1.5\hat{j} - a_{Bt} = 2\alpha\hat{j} + 2\alpha\hat{j}$$
$$- 0.25\hat{i}$$

$$2 + 1.5 = 2\alpha + 2\alpha \Rightarrow \quad \alpha = 0.875 \circlearrowleft$$

Now to find $(a_B, a_A)_n$

$$a_A = a_o + \alpha \times r_{A/o} - \omega^2 \times r_{A/o}$$

$$2\hat{j} + (a_n)_A\hat{i} = a_o\hat{j} + (0.875)\hat{k} \times 2\hat{i} - 0.125\hat{i}$$

$$2\hat{j} + (a_n)_A\hat{i} = a_o\hat{j} + 1.75\hat{j} - 0.125\hat{i}$$
$$(a_n)_A = -0.125 \quad \text{put in eq *}$$

$2\hat{j} + 0.125\hat{i} + 1.5\hat{j} - a_B\, t\, \hat{i} = \text{- - -}$

$-0.125 + (a_B)_t = -0.25$

$(a_B)_t = 0.125 \rightarrow$

So $V_B = -5 = 5\hat{j}$

$a_B = \sqrt{(0.125)^2 + (1.5^2)} = 1.505$

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* 16.8 : Used when you 2 points not on the same Rigid Body

→ C, A are not on the same Rigid body

$$\vec{V}_C = \vec{V}_A + \vec{\Omega} \times \vec{r}_{C/A} + \vec{V}_{C/A}$$

Ω: Angular velocity of the Coordinate System

$$\vec{a}_C = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{C/A} + \vec{\Omega} \times \vec{\Omega} \times \vec{r}_{C/A} + 2\vec{\Omega} \times \vec{V}_{C/A} + \vec{a}_{C/A}$$

$\dot{\Omega}$: Angular Acceleration of the coordinate System

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Example : 16-B3

$a_{C/B} = 0.5$ $v_{C/B} = 3$

$$\vec{V}_C = \vec{V}_B + \vec{\Omega} \times \vec{r}_{C/B} + \vec{V}_{C/B}$$

$$\vec{V}_C = 0 + (6\hat{k}) \times (0.5\hat{j}) + 3\hat{j}$$

$$\vec{V}_C = -3\hat{i} + 3\hat{j}$$

Now:-

$$\vec{a}_C = \vec{a}_B + \dot{\Omega} \times \vec{r}_{C/B} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{C/B}) + 2\Omega \times \vec{V}_{C/B} + \vec{a}_{C/B}$$

$$= 0 + (1.5\hat{k}) \times (0.5\hat{j}) + (6\hat{k}) \times (6\hat{k} \times 0.5\hat{j}) + 2 \times 6\hat{k} \times 3\hat{j} + 0.5\hat{j}$$

$$= -0.75\hat{i} + 6\hat{k} \times (-3\hat{i}) + -36\hat{i} + 0.5\hat{j}$$

$$= -0.75\hat{i} + -18\hat{j} - 36\hat{i} + 0.5\hat{j}$$

$$\vec{a}_C = -36.75\hat{i} - 17.5\hat{j}$$

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