Chapter 18 : Work & Energy

Kinetic Energy :-

$$\boxed{T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2}$$ For Every Type of Motion

$\frac{1}{2} m V_G^2$ = 0 If It's Rotation about a fixed axis

$\frac{1}{2} I_G \omega^2$ = 0 if It's only Translation

Work :-

1- variable force : $\boxed{U_F = \int_{S_1}^{S_2} F \cos\theta \, ds}$

2- Constant force : $\boxed{U_c = F \cos\theta \, S}$

3- Work of Weight $\boxed{U_w = -W \Delta y}$

4- Work of spring force $\boxed{U_s = -\frac{1}{2} k \left(S_2^2 - S_1^2\right)}$

5- Work of a Couple Moment $\boxed{U_M = \int_{\theta_1}^{\theta_2} M \, d\theta}$

- for Wheels : * No slipping → Friction does No Work
  * with " → " " Work

Principle of Work & Energy :-

$$\boxed{T_1 + \Sigma U_{1-2} = T_2}$$

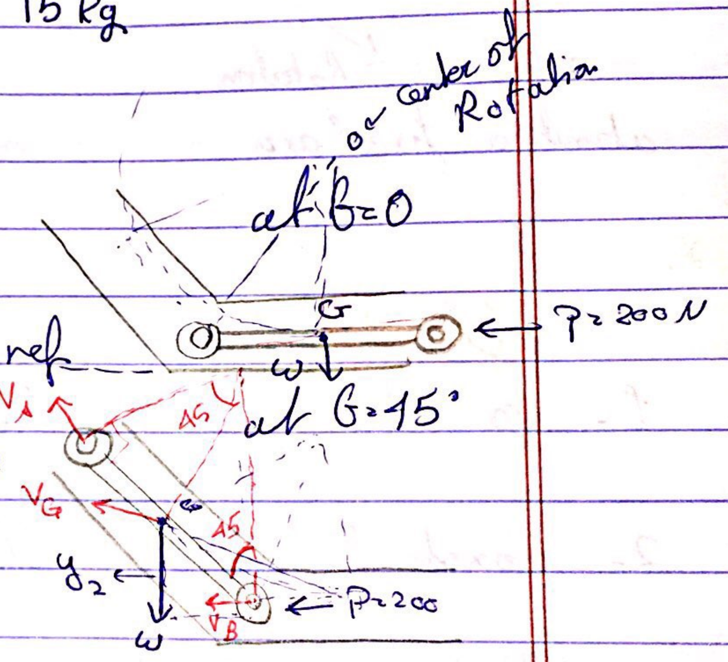
18-20

$P = 200\,N$ $\quad m_{slender} = 15\,kg$

at $\theta° = 0 \longrightarrow$ Rest

at $\theta° = 45$ $\quad \omega = ??$

$T_1 = 0$ (from rest)



$T_2 = \frac{1}{2} m_s V_G^2 + \frac{1}{2} I_G \omega^2$

$= (\frac{1}{2})(15)(\omega\, 0.6708)$

$+ (\frac{1}{2})\left((\frac{1}{12})(15)\, 0.6^2\right)\omega^2$

$= 3.599\,\omega^2$

$V_G = \omega\,(d_{G \to O})$

d_{G-O} :-

$d_{G.O} = \sqrt{0.3^2 + 0.6^2}$

$\Sigma U_{1-2} = U_w + U_p$

$= -(15)(9.81)\,(0.3\sin 45 - 0)$

$+ (200)(\cos 0)(0.6)$

$= 88.78$

$\sin 45 = \frac{y_2}{0.3}$

$T_1 + \Sigma U = T_2$

$\hookrightarrow \boxed{\omega = 4.97}$

• Conservation of Energy :-

Potential Energy:

1- Gravitational P.E : $\boxed{V_g = W y_G}$

above ref positive — under ref negative

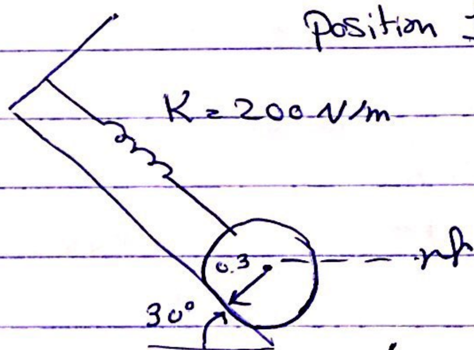
2- Elastic P.E :- $\boxed{V_e = \frac{1}{2} k s^2}$

↳ from 0 to S

$\boxed{T_1 + V_1 = T_2 + V_2}$ For the Whole System

18-47:

Position 1



$m = 10\ kg$

$V_1 = 4\ m/s$

$d = ??$ until $T_2 = 0$

ω.

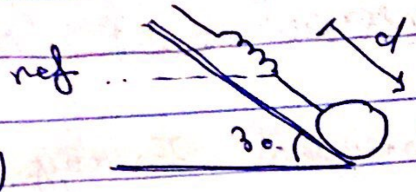
$$T_1 + V_1 = \cancel{T_2}^{0} + V_2$$

$$T_1 = (\tfrac{1}{2})(10)(16) + (\tfrac{1}{2})(\tfrac{1}{2})(40)(0.3)^2 \left(\frac{4}{0.3}\right)^2$$

~~$T_1 = 7?76$~~ $T_1 = 480$

$$V_1 = V_g + V_s = 0$$

at position 2:



$$V_2 = \left(\tfrac{1}{2}\right)(200)d^2 + -(40)(9.81)(d\sin 30)$$

$$= 100\,d^2 - 196.2\,d$$

$$480 = 100\,d^2 - 196.2\,d$$

$$d = \frac{+196.2 \pm \sqrt{(196.2)^2 - (4)(100)(-480)}}{(2)(100)}$$

$$d = \frac{196.2 + 480}{200}$$

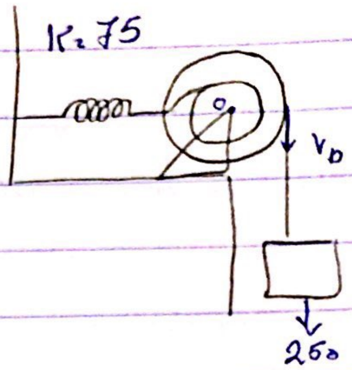
$$\boxed{d = 3.38}$$

18-54 $\qquad W = 250$ lb

V_b after descending 5 ft

$W_d = 50$ lb

$k_o = 0.5$ ft



1.071

$$T_1 + V_1 = T_2 + V_2$$

$T_1 = 0$ (from rest)

$V_1 = 0$ (sping unstretched, W on ref)

$$T_2 = \frac{1}{2} m_b V_b^2 + \cancel{\frac{1}{2} m_d V_G^2}^{\,0} + \frac{1}{2} I_G \omega^2 \leftarrow \left(\frac{V_b}{0.75}\right)^2$$

$$= \left(\frac{1}{2}\right)\left(\frac{250}{32.2}\right) V_b^2 + \left(\frac{1}{2}\right)\left(0.5^2\left(\frac{50}{32.2}\right)\right) \omega^2$$

$$T_2 = 4.227\, V_b^2$$

$S_b = r\theta$

$5 = (0.75)(\theta)$

$\theta = 6.67$

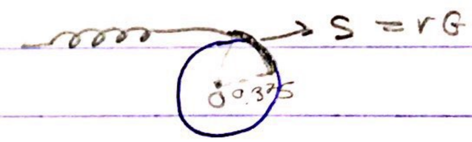
so $S_{\text{for spring}} = (0.375)(\theta)$

$= 2.5$

$$V_2 = V_g + V_e$$

$$= -(250)(5) + \left(\frac{1}{2}\right)(75)(2.5)^2$$

$$V_2 = -1015.625$$



$$0 = 4.227\, V_b^2 - 1015.625$$

$V_b = 15.5 \downarrow$