lec ①

Chapter 12: Kinematics of a particle

Kinematics ↓ Position/velocity acceleration/time

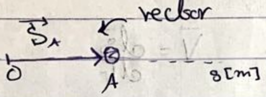
particle ↓ object with a Mass

no dimensions

- Transition only
- No Rotation

12-2. Rectilinear Kinematics : Continuous Motion

1) Position

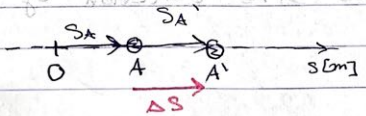


Def: Vector Quantity Representing the location of a particle with respect to an Origin

$|\vec{S}_A|$ = distance between O and the Particle.

Direction is arbitrary usually $\xleftarrow{-}\overset{+}{\uparrow}\xrightarrow{+}$ $\underset{-}{\downarrow}$

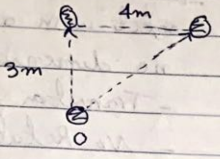
2) Displacement : vector



$\vec{\Delta S} = \vec{S}_{A'} - \vec{S}_A$

- In this Example:
- ΔS is positive when Motion is to the right.
- ΔS is negative when Motion is to the left

Difference Between distance and displacement



- $|disp| = 5m = 3i + 4\hat{j}$
- distance 7m

Unit : [m], [ft].

3) Velocity :-

a) Average Velocity $\vec{V}_{avg} = \frac{\Delta \vec{S}}{\Delta t}$

b) Instantaneous Velocity $\vec{V} = \frac{d\vec{S}}{dt}$

c) Speed : Magnitude of velocity $\vec{V}_{sp}$

d) Average speed $(V_{sp})_{avg} = \frac{S_T}{\Delta t}$

where S_T : total distance travelled.

Unit: [m/s] or [ft/s]

Note: Direction of $\vec{V}$ and $\vec{V}_{avg}$ is the same as the displacement (it shows the direction of Motion)

4) Acceleration

a) Average acceleration : $\vec{a}_{avg} = \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t}$

b) instantaneous Acceleration : $\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{S}}{dt^2}$

Unit : $[m/s^2]$ or $[ft/s^2]$.

- إشارة التسارع لا تدل على اتجاه الحركة.

- if $V_2 > V_1$ $\quad \vec{a}$ + (accelerating Motion)
- if $V_1 < V_2$ $\quad \vec{a}$ − (decelerating Motion)

Now:

$$\vec{V} = \frac{d\vec{S}}{dt} \rightarrow dt = \frac{d\vec{S}}{V}$$

$$\vec{a} = \frac{d\vec{V}}{dt}$$

$$\vec{a} = \frac{V\,d\vec{V}}{d\vec{S}}$$

$$\int_{S_1}^{S_2} \vec{a}\,ds = \int_{V_1}^{V_2} \vec{V}\,dv$$

lec 2:

· Constant Motion with Constant Acceleration.

1) Velocity :-

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\int_0^t \vec{a}_c \, dt = \int_{V_1}^{V_2} d\vec{v}$$

$$a_c t = V_2 - V_1$$

$$\boxed{V_2 = V_1 + a_c t}$$

2) Position as a function of time

$$\vec{V} = \frac{d\vec{S}}{dt}$$

$$\int_0^t \vec{V} \, dt = \int_{S_1}^{S_2} dS$$

$$\int_0^t V_1 + a_c t \, dt = S_2 - S_1$$

$$V_1 t + a_c \frac{t^2}{2} = S_2 - S_1$$

$$V_1 t + a_c \frac{t^2}{2} = \Delta S$$

$$\boxed{S_2 = S_1 + V_1 t + \frac{1}{2} a_c t^2}$$

3)

$$\vec{a} \, dS = \vec{V} \, dV$$

$$\int_{S_1}^{S_2} a_c \, dS = \int_{V_1}^{V_2} \vec{V} \, dV$$

$$a_c (S_2 - S_1) = \frac{V_2^2 - V_1^2}{2} \Rightarrow V_2^2 = V_1^2 + 2 a_c \Delta S$$

12.17 :-

- a particle
- straight line Motion.
- $a = 2t - 1$
- $\left.\begin{matrix} S = 1m \\ V = 2 m/s \end{matrix}\right\} t=0$

Determine S, V at $t = 6s$
S_T , total distance travelled.

$$a = \frac{dv}{dt}$$

$$a\,dt = dv$$
$$\int_0^t 2t-1\,dt = \int_2^v dv$$
$$t^2 - t = V - 2 \Longrightarrow V(t) = t^2 - t + 2$$

at $t = 6s$:-

$$\boxed{V = 32 \ m/s}$$

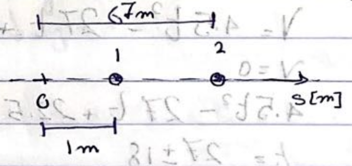
$$V = \frac{ds}{dt}$$

$$V\,dt = ds$$
$$\int_0^t t^2 - t + 2 = \int_1^s ds$$
$$\frac{t^3}{3} - \frac{t^2}{2} + 2t = S - 1$$

at $t = 6$:

$$\boxed{S = 67m}$$

Total distance travelled:-

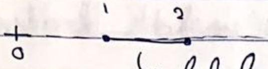


- اذا أراد الجسم تغير اتجاهه لابد أنه يتوقف لذلك نجد متى $V=0$. ونرى اذا موجود في الفترة الزمنية المعطى

$$V(t) = t^2 - t + 2 = 0 \Rightarrow t = \frac{1 \pm \sqrt{-7}}{2} \text{, Complex}$$

- اذا من المعادلة سرعته لم تصبح صفر معناها الجسم مشي بخط مستقيم

e)



↳ total distance travelled.

$S_T = 67-1 = 66m$

12.10 :-

$S = 1.5t^3 - 13.5t^2 + 22.5t$ ft

Find S

S_T when $t = 6s$

$\boxed{S = -27}$

$S_T = ?$

Find V :-

$\frac{ds}{dt} = V$

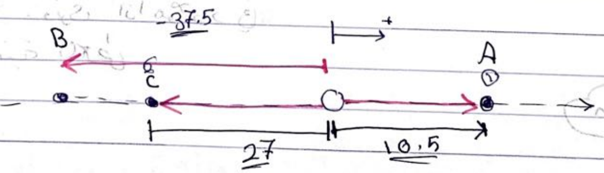
$V = 4.5t^2 - 27t + 22.5$

$V = 0$

$4.5t^2 - 27t + 22.5 = 0$

$t = \frac{27 \pm 18}{9}$

$t_1 = 5$ s $t_2 = 1$ s



at $t = 1$

$S = 1.5 - 13.5 + 22.5 = 10.5$ ft with Respect to the Origin.

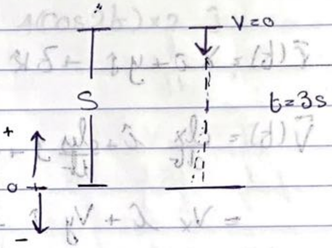
at $t=5$,

$S = -375$ ft

$A \rightarrow B \rightarrow C$

$S_T = 10.5 + 10.5 + 48 = 69$ ft

12.9:-



Find S, V_2

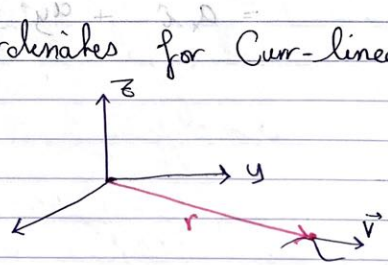
$g = 9.81$ m/s = 32.2 ft/s
Since $a=g$

$V_2 = V_1 + at$
$V_2 = 0 - (9.81)(3)$
$V_2 = -29.4$ m/s

if $\uparrow + \; g \uparrow = -9.81$
if $+\downarrow \; g = 9.81$

$\Delta S = V_1 t + \frac{1}{2} a t^2$
$-S = 0 - \frac{(9.81)}{2}(9)$
$\boxed{S = 44.1}$

12.5 : Rectangular Coordinates for Curr-linear Motion

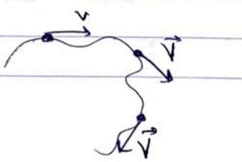


$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$
$\vec{v}(t) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$

$= V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$

$|\vec{v}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$

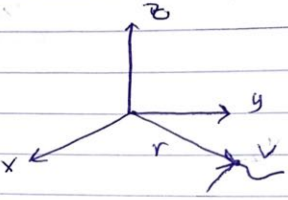
it is always tangent to the Curve



$$\vec{a} = \frac{d^2x}{dt^2}\,\hat{i} + \frac{d^2y}{dt^2}\,\hat{j} + \frac{d^2z}{dt^2}\,\hat{k}$$

$$= a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$$

S. 12-5 : Rectangular Coordinates for Curvilinear Motion



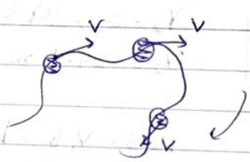
$$\vec{r}(t) = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

$$\vec{v}(t) = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} + \frac{dz}{dt}\,\hat{k}$$

$$= v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$\hat{it}$ is always tangent to the Curve



$$\vec{a}(t) = \frac{d^2x}{dt^2}\,\hat{i} + \frac{d^2y}{dt^2}\,\hat{j} + \frac{d^2z}{dt^2}\,\hat{k}$$

$$= a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$$

Dynamics 3:-

12-73:

$\vec{r} = (5\cos 2t)\hat{i} + (4\sin 2t)\hat{j}$ m

Find $|\vec{v}|$

$|\vec{a}|$ at $t = 1s$

$$\vec{V} = \frac{d\vec{r}}{dt} = (5)(-\sin 2t)\times 2\ \hat{i} + 4(\cos 2t)\times 2\ \hat{j}$$

$\vec{V} = -10\sin 2t\ \hat{i} + 8\cos 2t\ \hat{j}$

V at t=1 :-

$\vec{V} = -10\sin 2\ \hat{i} + 8\cos 2\ \hat{j}$

$$\left.\begin{array}{l}\pi \text{ rad} = 180^\circ \\ 2 \text{ rad} = ?\end{array}\right\} \qquad \boxed{\theta = \frac{2\times 180}{3.14159}} =$$

$\vec{V}(1) = -9.093\ i - 3.329\hat{j}$

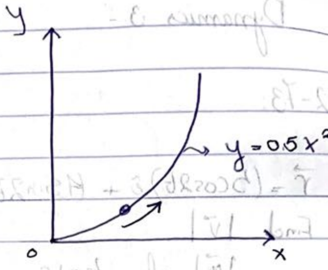
① $|\vec{V}| = \sqrt{(-9.093)^2 + (3.329)^2} = 9.68$ m/s

$\vec{a} = -20\cos 2t\ \hat{i} - 16\sin 2t\ \hat{j}$

$\vec{a}(1) = 8.323\ \hat{i} - 14.549\hat{j}$

② $|\vec{a}| = 16.8$ m/s²

12-83 :



$V_x = 5t$ ft/s

Find $|\vec{r}|$

$|\vec{a}|$ when t=1 s

where at $t = 0$, $x = 0$
$y = 0$

$V_x = \frac{dx}{dt}$

$dx = V_x \, dt$

$\int_0^x dx = \int_0^t 5t \, dt$

$X = \frac{5t^2}{2} \Rightarrow \boxed{x = 2.5t^2}$

$y = 0.5 \, (2.5t^2)^2$

$\boxed{y = 3.125t^4}$

X at 1 = 2.5 ft

y at 1 = 3.125 ft

so $|\vec{r}| = \sqrt{(2.5)^2 + (3.125)^2} = 4.$ ft

$a_x = \frac{dv_x}{dt} = 5$ ft/s²

$v_y = 12.5t^3$

$a_y = 37.5t^2$

$a_y = 37.5$ ft/s² $\Rightarrow |\vec{a}| = \sqrt{(37.5)^2 + (5)^2} = 37.8$ ft/s²

• Another Method to solve Question:-

$y = 0.5\,x^2$ derive with Respect to t

$$v_y = \dot{y} \leftarrow \frac{dy}{dt} = \overset{\dot{x}}{\frac{dx}{dt}}\ \overset{x}{(x)}$$

$$\frac{d^2y}{dt^2} = \frac{dx}{dt}\frac{d(x)}{dt} + x\,\frac{d^2x}{dt^2}$$

$$a_y = \left(\frac{dx}{dt}\right)^2 + x\,\frac{d^2x}{dt^2}$$

$$\boxed{a_y = V_x^2 + x\,a_x}$$

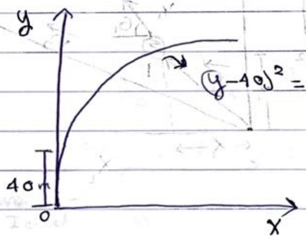
• ملاحظة هامة :
الاشتقاق بالنسبة لـ t
وليس x

12.86 :

$V_y = 180$ m/s $\overset{*}{(\text{constant})}$

Find $|\vec{V}|$

$|\vec{a}|$ at $y = 80$m



$* \boxed{a_y = 0}$

$$(y-40)^2 = 160\,x$$

$$2(y-40)\frac{dy}{dt} = 160\frac{dx}{dt}$$

$$2(y-40)\,\dot{y} = 160\,\dot{x} \quad (\Longleftarrow) \quad \boxed{2(y-40)V_y = 160\,V_x}$$

Derive wr to t:

$$2(y-40)\,\ddot{y} + \dot{y}\,\big(2(\dot{y})\big) = 160\,\ddot{x}$$

$$\boxed{2(y-40)\,\ddot{y} + 2\dot{y}^{2} = 160\,\ddot{x}}$$

put $V_y = 180$

$V_x = 90$ m/s

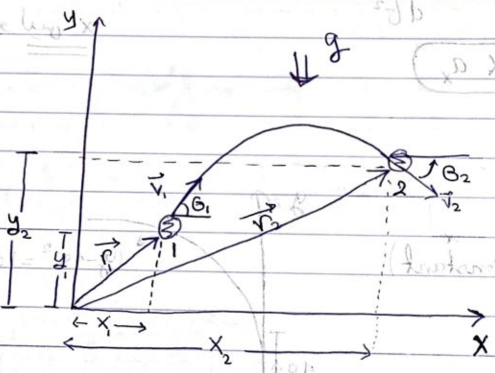
$|\vec{V}| = \sqrt{90^2 + 180^2} = 201$

$$\boxed{2\left[(y-40)a_y + V_y^2\right] = 160\, a_x}$$

So $\boxed{a_x = 405\ m/s^2}$

$|\vec{a}| = 405\ m/s^2$

12.6: Projectile Motion



• ملاحظة هامة
• السرعة دائماً مماس
لل curve

* Horizontally: $\xrightarrow{+}$

Prove ① $V_2 = V_1 + a_c t$

$V_{2x} = V_{1x} + 0$

So: V_x is Constant • الحركة الأفقية للسرعة ثابتة

Prove ② $V_2^2 = V_1^2 + 2a_c(s_2 - s_1)$

$V_{2x}^2 = V_{1x}^2 + 0$

$V_{2x} = V_{1x}$

$\Delta S = V_1 t + \frac{1}{2} a_c t^2$

$\boxed{X_2 = X_1 + V_{1x} t} \rightarrow$ • معادلة حركة نستخدمها

Verbically: ↑+

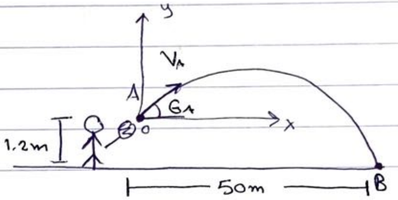
$V_{2y} = V_{1y} \ominus g t$ — ①

$V_{2y}^2 = V_y^2 \ominus 2g(y_2 - y_1)$ — ②

$y_2 = y_1 + V_{1y} t \ominus \frac{1}{2} g t^2$ — ③

↑+

• على نفس الارتفاع تكون V_{y_1} تساوي V_{y_2} مقدارًا، وتعاكسها اتجاهًا



12-89.

$t_{AB} = 2.5$ s

$V_x = \frac{500}{2.5} = 20$ m/s

$V_{y_B} = V_{y_A} + a_y t$

$\boxed{V_{Ax} = 20 \text{ m/s}}$

$V_{Ax} = V_A \cos\theta_A$ — ①

$V_{y_B} = V_{y_A} - (9.81)(2.5)$

~~zero~~ I dont know if

so: $y_2 = y_1 + V_{1y} t - \frac{1}{2} g t^2$

$-1.2 = 0 + V_{Ay}(2.5) - \frac{1}{2}(9.81)(2.5)^2$

$\boxed{V_{Ay} = 11.78}$

$V_{Ay} = V_A \sin\theta_A$ — ②

Divide 2 by 1: $\frac{V_{Ay}}{V_{Ax}} = \tan\theta_A$

$\frac{11.78}{20} = \tan\theta_A$

$\boxed{\theta_A = 30.5^\circ}$

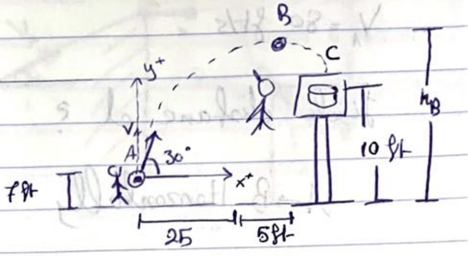
so $V_A = \frac{20}{\cos(30.5)} = 23.2$ ✓

Dynamics

lec 3

12-99 :-

Find V_A , h_B



A → C Horizontally

$$X_C = X_A + V_{Ax} t_{AC}$$
$$30 = 0 + V_{Ax}\ t_{AC}$$
$$30 = V_A \cos 30\ t_{AC} \quad \text{——} \ ①$$

A → C vertically

$$y_C = y_A + V_{Ay} t_{AC} - \frac{1}{2} g t_{AC}^2$$

$$3 = 0 + V_A \sin 30\ t_{AC} - \frac{32.2}{2}\ t_{AC}^2$$
$$3 = V_A \sin 30\ t_{AC} - 16.1\ t_{AC}^2 \ \text{——} \ ②$$

Solving ① with ②

$t_{AC} = 0.943$ s $\qquad V_A = 36.7$ ft/s

Now to find h_B :-

Horizontally A → B

$$X_B = X_A + V_{Ax} t_{AB}$$

$$25 = 0 + (36.7)(\cos 30)\ t_{AB}$$
$$t_{AB} = 0.786\,s$$

Vertically A → B :- $\quad y_B = y_A + V_{Ay} t_{AB} - \frac{1}{2} g t_{AB}^2$

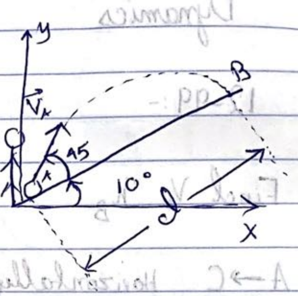
عن المرجع (مستوى الكرة)

$$h_B - 7 = 0 + (36.7) \sin 30 (0.786) - (\tfrac{1}{2})(32.2)(0.786$$
$$h_B = 11.5 \text{ ft}$$

12-102

$V_A = 80$ ft/s

find distanc d ?



A→B Horizontally

$$X_B = X_A + V_{Ax} t_{AB}$$
$$d\cos 10 = 0 + (80 \cos 55)(t_{AB})$$
$$d \cos 10 = 80 \cos 55 \; t_{AB} \quad \text{——} ①$$

A→B vertically

$$y_B = y_A + V_{Ay} t_{AB} - \frac{1}{2} g t_{AB}^2$$

$$d \sin 10 = 0 + 80 \sin 55 \; t_{AB} - \frac{32.2}{2} t_{AB}^2 \quad \text{——} ②$$

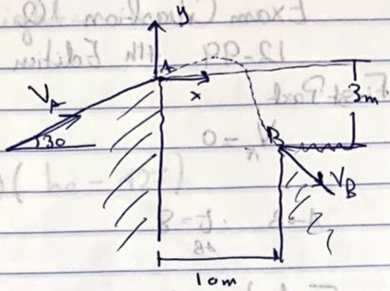
Solving ① with ②

$t_{AB} = 3.57$ s

$d = 166$ ft

Exam Question ①

Find V_A, V_B, G_B



Horizontally:-

$X_B = X_A + V_{Ax} t_{AB}$

$10 = 0 + V_A \cos 30 \, t_{AB}$ — ①

vetrically

$y_B = y_A + V_{Ay} t_{AB} - \frac{1}{2} g t_{AB}^2$

$-3 = 0 + V_A \sin 30 \, t_{AB} - \frac{1}{2} (9.81) t_{AB}^2$ — ②

$t_{AB} \quad V_A$ ✓

✓ $V_{Bx} = V_{Ax} = V_A \cos 30$

$V_{By} = V_{Ay} - g t_{AB}$

✓ $V_{By} = V_A \sin 30 - (9.81) t_{AB}$ (negative)

$V_B = \sqrt{V_{By}^2 + V_{Bx}^2}$

$G_B = \tan^{-1} \frac{V_{By}}{V_{Bx}}$ (negative) 4th Quarter

Exam Question ②

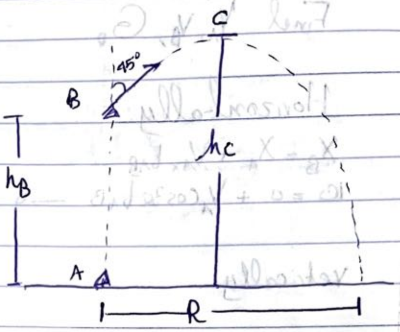
12-99 4th Edition

- First Part

$V_A = 0$

$A \to B \quad : t_{AB} = 8$

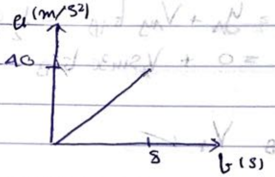
Find h_B , V_B



- Second PART:-

$B \to C$

Free Flight : Projectile/a is Constant



Find h_C , R

Now: $A \to B$ Rectilinear Motion Under varrying acc

$a = \frac{dv}{dt} = 5t$ ميل الخط

$\hookrightarrow \quad y = ax + b$

ميل المقطع الصادي

$a = \frac{dy}{dt}$

$\int_0^t a\,dt = \int_0 dv$

$\frac{5t^2}{2} = V \Rightarrow V = 2.5t^2$

at $t = 8 \to V = (2.5)(8)^2 = 160$

$V = \frac{ds}{dt} \Rightarrow \int_0^{vt} V dt = \int_0^s ds \Rightarrow \frac{2.5t^3}{3} = S$

put $t = 8$:- $S = 426.7$ m

Now from B → C   Free flight :-

$$V_{C_y}^2 = V_{B_y}^2 + 2a\,(h_C - h_B)$$

$$0 = (160\cos 45)^2 + (-2)(9.81)(\,h_C - 427)$$

$$\frac{12800}{(2)(9.81)} = h_C - 427$$

$$h_C = 1079.4$$

B → D Horizontally

$$X_D = X_B + V_x\, t_{AD}$$

$$R = 0 + V_x\, t_{BD} \quad \text{—} ①$$

B → D verticaly

$$-h_B = 0 + (160)\cos 45\; t_{BD} - (\tfrac{1}{2})(9.81)\, t_{BD}^2$$

$$-427 = 0 + \text{———} \quad \text{—} ②$$

find t_{BD}

$t_{BD} = 26.36\,s$

$$R = (160)(\sin 45)(26.36)$$

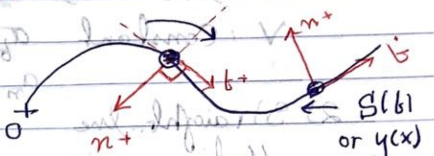
$$R = 2983\ m$$

Dynamics — lec 5

* 12.7: Curvilinear Motion: Normal Tangential Coordinats

tangential — t-axis: tangential to the Curve positive with the direction of Motion. ($\vec{U}_t$) ↳ unit vector

الموجب مع
اتجاه الحركة



Normal — n-axis: ⊥ t-axis

positive towards the Center of the Curve ($\vec{U}_n$)

binormal → b-axis: ⊥ t-axis and ⊥ n-axis ($\vec{U}_b$)

positive direction is found using Right hand Rule.

1) Velocity :-

$$\vec{V} = \underset{\text{magnitude}}{v}\,\vec{U}_t$$

• velocity is always tangent to the Curve.

مشتقة الموضع بالنسبة للزمن $= \dot{S}\,\vec{U}_t$

2) Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \underbrace{v\,\dot{\vec{U}}_t} + \dot{v}\,\vec{U}_t$$

• $\dot{U}_t = \frac{v}{\rho}\,\vec{U}_{normal}$

$$= \frac{v^2}{\rho}\overset{normal}{\vec{U}_n} + \dot{v}\,\vec{U}_t$$

ρ: Radius of Curvature

$$\boxed{\vec{a} = \ddot{S}\,\vec{U}_t + \frac{\dot{S}^2}{\rho}\,\vec{U}_n}$$

a_t (tangential acc) — a_n (normal acc)

$$|\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

a_n: لا تساوي الصفر إلا للجسم
الساكن

Notes:

1) • Rectilinear Motion

$\dot{v}$: constant $\vec{a} = 0$

• Curvilinear Motion

V : Constant $a_t = 0$

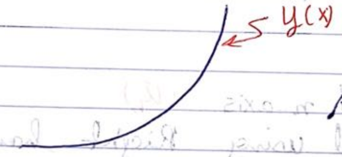
$a_n = \frac{V^2}{\rho}$

2) Straight line Motion

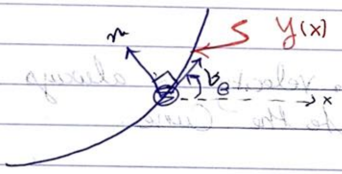
$\rho = \infty$ so $a_n = 0$

$a_t = \dot{V}$

Ex:



$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$



$$^{*}\ \theta_{xt} = \tan^{-1} \frac{dy}{dx}$$

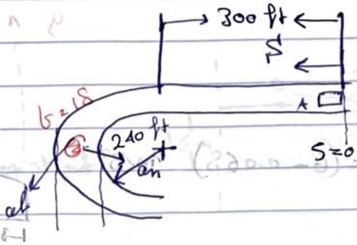
12-119 :-

$V_A = 0$

$\dot{V} = 0.05t^2$ ft/s²

Find $|\vec{v}|$

$|\vec{a}|$ at $t = 18$ s



$$\int_0^V dv = \int_0^t \dot{v}\,dt$$

$$\boxed{V = \frac{0.05t^3}{3}}$$

$$\int_0^S ds = \int_0^t V\,dt$$

$$S = \frac{0.05t^4}{12}$$

$$S_{at\,t=18} = \frac{0.05(18)^4}{12} = 437.4 \text{ ft}$$

- The car is in the Curved Region after 18 s

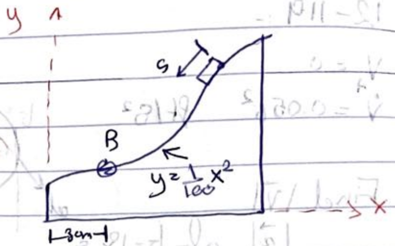
$$V(18) = \frac{0.05(18)^3}{3} = 97.2 \text{ ft/s}$$

$$a_t = (0.05)(18)^2 = 16.2 \text{ ft/s}^2$$

$$a_n = \frac{V^2}{\rho} = \frac{(97.2)^2}{240} = 39.37 \text{ ft/s}^2$$

$$\vec{a} = 42.6 \text{ ft/s}^2$$

12-122 :-



$V_A = 0$

$a_t = \dot{V} = (6 - 0.06S)$ m/s^2

$S_B = 40$ m

Find $|\vec{a}_B|$

$a_{t_B} = 6 - 0.06 S_B$

$= 6 - 0.06(40)$

$= 3.6$ m/s^2

$(\vec{a}_{n_B}) = \frac{V^2}{\rho} = \frac{2[6(40) - 0.03)(40)^2]}{79.3}$

$\int_0^s a_t \, ds = v \, dv$

$\int_0^s 6 - 0.06S \, ds = \int_0^v v dv$

$6S - \frac{0.06S^2}{2} = \frac{V^2}{2}$

$6S - 0.03S^2 = \frac{V^2}{2}$

$\rho_B = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$

$= \left[\frac{\left(1 + \left(\frac{1}{50}x\right)^2\right)}{\left(\frac{1}{50}\right)}\right]^{\frac{3}{2}}$

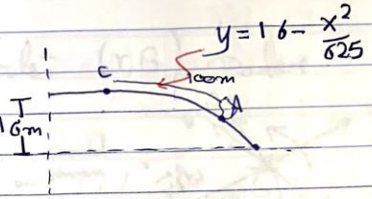
$= \left[1 + \left(\frac{30}{50}\right)^2\right]^{\frac{3}{2}} \times 50$

$\rho_B = 79.3$m

$\vec{a}_{n_B} = \frac{(19.6)^2}{79.3} = 4.84$ m/s^2

$|\vec{a}_B| = 6.03$ m/s^2

12- 126 :-



$V_A = 20$ m/s

$a_t = 0.5$ m/s² .... constant

Find $|\vec{a}_c|$

$S_c = 100$m

$(a_t)_c = (a_t)_A = 0.5$ m/s² Constant

$$V_C^2 = V_A^2 + 2a_t(S_c - S_A)$$
$$V_c^2 = (20)^2 + 2(0.5)(100 - 0)$$
$$= 400 + 100 = 500$$

X = 0 when y = 16

$V_C^2 = 500$

$$\rho_c = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \left(\frac{-2x}{625}\right)^2\right]^{3/2}}{\left|\frac{-2}{625}\right|} = \frac{\left[1 + \left(\frac{-2 \times \cancel{100}^{0}}{625}\right)^2\right]^{3/2}}{\frac{2}{625}}$$

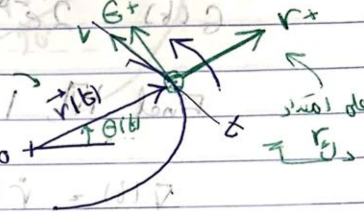
$$\rho_c = \frac{625}{2} = 312.5 \text{ m}$$

So $(a_n)_c = \frac{500}{312.5} = 1.6$ m/s²

$|a_c| = 1.68$ m/s²

lec 6

12.8: Cylindrical Coordinates (r, θ) Polare Coordinats.



This should be known.

- r-axis :-
Positive: in the direction of increasing r if θ is Constant $\vec{u}_r$

- θ-axis :- ⊥ r-axis in the direction of increasing θ $\vec{u}_\theta$

* ملاحظة: اتجاه θ ليس مهم، الا اذا كان عشانا دائرة.

- z-axis : ⊥ r-axis, ⊥ θ-axis, following the right hand rule. $\vec{u}_z$

$$\vec{r}(t) = r\,\vec{u}_r$$

① Velocity :-

$$\vec{V}(t) = \dot{r}\,\vec{u}_r + \dot{\vec{u}}_r\, r \longrightarrow = \overset{\text{rad/s}}{\dot{\theta}}\,\vec{u}_\theta$$

$\dot{\theta}$: rate of change of the angle with respect to the time

$$\boxed{\vec{V}(t) = \underbrace{\dot{r}\,\vec{u}_r}_{v_r} + \underbrace{r\,\dot{\theta}\,\vec{u}_\theta}_{v_\theta}}$$

→ المحصلة اتجاهها باتجاه ال tangent.

② Acceleration :- $\dot{\theta}\,\vec{u}_\theta$

$$\vec{a}(t) = (\ddot{r}\,\vec{u}_r + \dot{r}\,\dot{\vec{u}}_r) + r\,\dot{\theta}\,\underset{-\dot{\theta}\vec{u}_r}{\dot{\vec{u}}_\theta} + [r\,\ddot{\theta} + \dot{r}\,\dot{\theta}]\,\vec{u}_\theta$$

$$=$$

$$\vec{a}(t) = \underbrace{[\ddot{r} - r\,\dot{\theta}^2]\,\vec{u}_r}_{\vec{a}_r} + \underbrace{[r\,\ddot{\theta} + 2\dot{r}\,\dot{\theta}]\,\vec{u}_\theta}_{a_\theta}$$

محصلتهم مش ضروري تكون مماس.

12–1661

$\vec{r}(t) = 4(1+\sin t)$ m

$\theta(t) = 2e^{-t}$ rad

Find $|\vec{V}|$, $|\vec{a}|$ when $t = 2s$:-

$\vec{V}(t) = \dot{r}\vec{U}_r + r\dot{\theta}\vec{U}_\theta$

$=$

$\dot{r}(t) = 4(\cos t)$ $\quad r = 4(1+\sin 2)$

$\dot{\theta}(t) = -2e^{-t}$

$\vec{V}(t) = 4\cos 2 + (4+4\sin 2)(-2e^{-2})$

$\vec{V}(2) = [-1.665\vec{u}_r + (7.623)(-0.2707)]\vec{u}_\theta = -2.07$ m/s

$\boxed{\vec{V}(2) = -2.07}$

$\ddot{r}(t) = -4\sin t =$

$\ddot{\theta}(t) = 2e^{-t} =$

$\vec{a}(t) = -4\sin t - r \ldots$

$\boxed{\vec{a}(t) = -4.2u_r + 2.97u_\theta}$

$=$

Notes

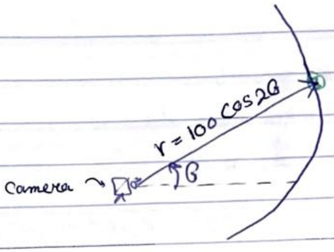
$X = 4\theta$

Radians لـ

Calculator θ بعدين Degrees تحول

$X = 2\sin\theta$ دخلها

12- 176 :-



$|\vec{V}| = 40$ m/s

$\dot{\theta}$? when $\theta = 15°$

$$|\vec{v}| = \sqrt{V_r^2 + V_\theta^2}$$

$$\vec{V}_r = \dot{r}$$
$$\vec{V}_\theta = r\dot{\theta}$$

Now: $r = 100\cos 2\theta = 86.6$ m

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt}$$
$$= -(100)(2)\sin 2\theta)(\dot{\theta})$$
$$= -200\sin 2\theta \cdot \dot{\theta}$$
$$\dot{r} = -100\dot{\theta}$$

$$\vec{V}_r = -100\dot{\theta}$$
$$\vec{V}_\theta = 86.6\dot{\theta}$$

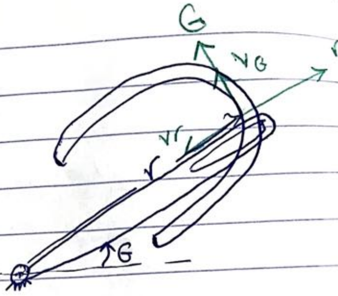
$$40 = \sqrt{(-100\dot{\theta})^2 + (86.6\dot{\theta})^2}$$
$$\dot{\theta} = 0.302 \text{ rad/s}$$

- we took the plus since the movement is clockwise

12-180

$\vec{r} = 4 \sin 2\theta$ ft

$\dot{\theta} = 1.5$ rad/s Constant



Find $|\vec{v}|$, $|\vec{a}|$ at $\theta = 60°$

$\dot{\theta} = 1.5$ rad/s

$\ddot{\theta} = 0$ rad/s²

* Chain Rule :-

$\dot{r} = \frac{dr}{dt} = 8 \cos 2\theta \times \dot{\theta}$

$\dot{r} = 8 \cos 2 \times 60 \times (1.5)$

$\dot{r} = -6$ ft/s

$\ddot{r} = 8[\cos 2\theta . \ddot{\theta} + \dot{\theta}(-2 \sin 2\theta . \dot{\theta})]$

$\ddot{r} = -31.18$ ft/s

$\boxed{r = 3.464 \text{ ft}}$

$V_r = -6$ ft/s

$V_\theta = 5.196$ ft/s

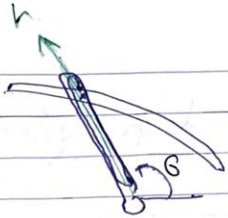
$V = 7.94$ ft/s

$a_r = -38.97$ ft/s²

$a_\theta = -18.$ ft/s²

$a = 42.9$ ft/s²

12-184



$r = 1.5\,\theta$

The arm starts from rest at $\theta = 60°$

$\dot{\theta} = 4t$ rad/s

Find : $v_r, v_\theta, a_r, a_\theta$ at $t = 1$ s

$\dot{\theta} = (4)(1) = 4$ rad/s
$\ddot{\theta} = 4$

$r = 1.5\,\theta$
$\dot{r} = 1.5\,\dot{\theta} \rightarrow \dot{r} = (1.5)(4) = 6$ m/s
$\ddot{r} = 1.5\,\ddot{\theta} \qquad \ddot{r} = (1.5)(4) = 6$ m/s

$\cancel{\int_{\theta_0}^{\theta} \dot{\theta} = \int_0^{t} 4t}$

$\dot{\theta} = \frac{d\theta}{dt}$

$\int_0^t \dot{\theta}\, dt = \int_{\pi/3}^{\theta} d\theta$

$\int_0^t 4t\, dt = \theta - \frac{\pi}{3}$

$\frac{4t^2}{2} = \theta - \frac{\pi}{3}$

$2 = \theta - \frac{\pi}{3} \rightsquigarrow \theta = 2 + \frac{\pi}{3} = \frac{6+\pi}{3} = 3.05$ rad
$= 174.6°$

$r = (1.5)(3.05) = 4.571$ ft

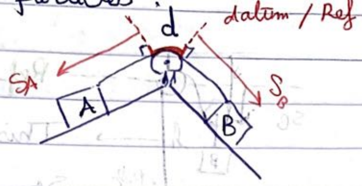
تعويض مباشر

$a_r = -67.1$ ft / $V_r = 6$ ft/s
$a_\theta = 66.3$ ft/s^2 / $V_\theta = 18.3$ ft/s

①

lec 7

12.9 Absolute Dependant Motion Analysis of two particles.



1) Pulley is frictionless
2) Cord is inextensible (length is Constant)

$L = S_A + d + S_B$

$\frac{d}{dt}\left(L = S_A + d + S_B \right)$

$0 = V_A + 0 + V_B$

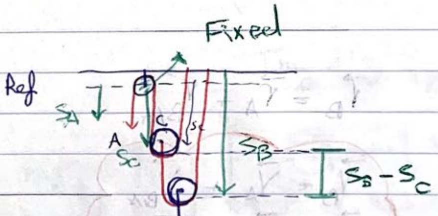
$$\boxed{\begin{aligned} V_A &= -V_B \\ a_A &= \underset{\text{Cord is increasing or decreasing in length (Direction)}}{\underbrace{-}} a_B \end{aligned}}$$

• الحبل تقصر من جهة (−)
و يطول من جهة أخرى

12-199 :-

$V_A = 2\,m/s$

Find V_B



$L_1 = S_A + S_C + S_C$

$\boxed{0 = V_A + 2V_C}$ — ①

• لكي نسمي قطعة حبل يجب ان يكون لها علاقة بـ Ref

$L_2 = S_B - S_C + S_B$

$\boxed{0 = 2V_B - V_C}$ — ②

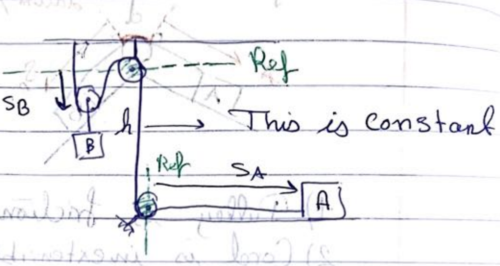
$V_C = -\frac{V_A}{2} = -1$

$0 = 2V_B + 1$

$\boxed{V_B = \underset{\text{B تتحرك لأعلى والحبل يقصر}}{\underbrace{-}}0.5}$

Example:-

Find the Relationship Between V_A + V_B, a_A + a_B



$L = S_B + S_B + S_A$

$0 = V_B + V_B + V_A$

$\boxed{0 = 2V_B + V_A}$

• أي حبل يجمع بين بكرتين مثبتين يكون طوله ثابت

① : $V_A = -2V_B$

$a_A = -2a_B$

• إذا طولت A فان B تقصر والعكس صحيح

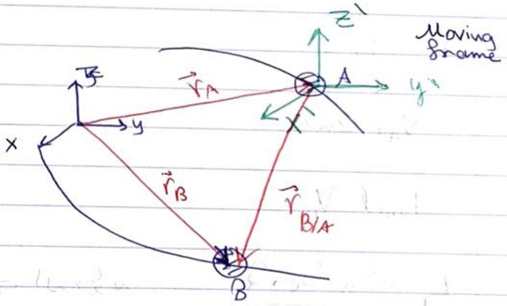
12.10 Relative Motion Using translating

- Fixed $\xrightarrow{\text{sees}}$ Absolute Motion تكون بالنسبة لمرجع ثابت
- In the Moving object $\xrightarrow{\text{sees}}$ Relative Motion 〃 〃 〃 متحرك

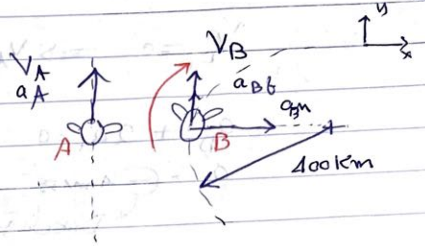
$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$



Example:-



$V_A = 700$ Km/h

$V_B = 600$ Km/h

$\vec{\dot{V}}_A = 50$ Km/h

$\vec{\dot{V}}_B = -100$ Km/h

Find $\vec{V}_{B/A}$ $\vec{a}_{B/A}$

$V_B = \vec{V}_A + \vec{V}_{B/A}$

$600\hat{j} = 700\hat{j} + \vec{V}_{B/A}$

$\vec{V}_{B/A} = -100\hat{j}$ Km/h

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

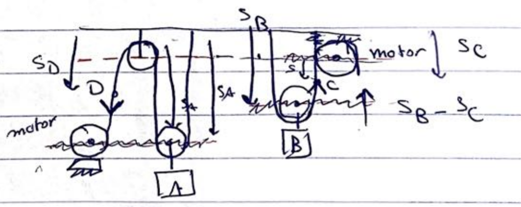
$-100\,\hat{j} + \left(\frac{V^2}{400}\right)\hat{i} = 50\,\hat{j} + \vec{a}_{B/A}$

$-100\,\hat{j} +$ ———

$\vec{a}_{B/A} = 900\,\hat{i} - 150\hat{j}$

Example:- Exam Question:-

The Motor is not a pulley



$V_C = 4$ m/s ↑ constant

$a_D = 8$ m/s² Constant

$V_D = 0$ at $t = 0$

Find : 1) t needed for A to Rise 3 meters

2) $\vec{V}_{B/A}$

$L = S_D + 2S_A \rightarrow V_D + 2V_A = 0$ — ①

$V_{D_1} = 0 \Rightarrow V_{A_1} = 0$

$a_D + 2a_A = 0$

$a_A = \underbrace{-}_{\text{Direction}} 4 \, m/s^2$

$S_{A_2} = S_{A_1} + V_{A_1} t + \frac{1}{2} a_A t^2$

$-3 = 0 + 0 + (\frac{1}{2})(-4)(t^2)$

$\frac{6}{4} = t^2$

$\boxed{t = 1.22 \, s}$

2) $\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$V_{A_2} = V_{A_1} + a_A t$

$V_{A_2} = 0 + (-4)(1.22)$

$\uparrow V_{A_2} = -4.88$

$-2 = -4.88 + V_{B/A}$

$V_{B/A} = -2 + 4.88 = 2.88$

$L = S_B + S_B - S_C$

$0 = (2S_B)' - (S_C)'$

$0 = 2V_B - V_C$

$2V_B + 4 = 0$

$\boxed{V_B = -2} \quad \uparrow$