lec 8

Dynamics

Kinematics
study of Motion
(vel, acc, position)
without taking into
Consideration any
Forces
Ch 12/16

Kinetics
study of forces Causing
the motion

- Force & Acc
  Newton's 2nd law
  Ch 13/17
- Work & Energy
  Ch14/18
- Impulse and Moment
  Ch 15/19

Chapter 13: Kinetics of a Particle: Forces and Acceleration

13.1 Newton's second law of Motion

" When an unbalanced force acts on a particle, The particle accelerates in the direction of the force, with a magnitude That is proportional to the force. "

Equation of Motion EOM $\rightsquigarrow \sum \vec{F} = \underset{\text{Kg}}{m}\overset{m/s^2}{\vec{a}}$ or [slug] [ft/s²]

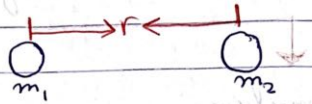
$\boxed{Kg \cdot m/s^2 = N}$

$\boxed{slug \cdot ft/s^2 = Ib}$

Mass:- [m] Positive Scalar
↳ Quantitative measure of the Resistance of a Particle to a $\underbrace{\text{Change in it's velocity}}_{\text{Acceleration}}$

- Newton's law of Gravitational attraction

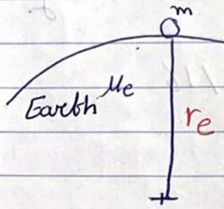
$$F_{attraction} = \frac{G m_1 m_2}{r^2}$$



$$G = 66.73 \times 10^{-12} \; m^3/Kg.s^2$$

$$F_{\substack{attraction\\earth}} = \frac{(\boxed{G})(M_e)(m)}{\boxed{r_e^2}}$$

Constant
Constant
Constant



$$\boxed{W = 9.81 \; m}$$

weight

m/s²
Gravatational
Accerlaration

$1 \; m = 3.28084 \; ft$

$9.81 \; m/s^2 = 32.185 \; ft/s^2$

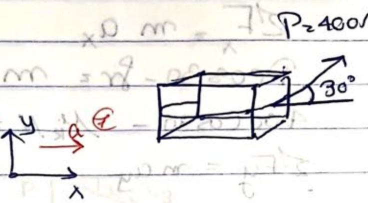
$$\boxed{g = 32.2 \; ft/s^2}$$

* Steps to Solve Kinetic Problems:-

1) Draw Free Body Diagram FBD
2) Solve for Unknowns
3) Link to the Other Kinematic Quantites ~~are~~ as required.

*1 Drawing the FBD

Example:-
$m = 50$ Kg
starts from rest
$\mu_k = 0.3$
Find V after 3s



[1] Set a frame of reference - one axis in the direction of motion.

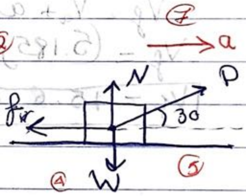
* Rectilinear Motion : x, y, z
* Curvilinear Motion : t, n, b or r, θ, z

[2] Isolate the Object of interest - Draw out-line of the object.

[3] Sketch all external Forces ②
- Active forces and Reactive forces.

المسببة الحركة ↗ ↗ تمنع الحركة باتجاهها

e.g. pin/ tension/ Roller / Normal



اتجاه الـ Friction بعكس اتجاه الحركة

[4] Do Not forget the weight.

[5] label all forces with magnitude and direction.

[6] Direction of Unknow Forces is assumed

[7] Set Direction of Acceleration
- assume +ve if unknown

[8] write equations of Motions and solve for Unknowns

$\Sigma F_x = m\,a_x$

$P\cos 30 - fr = m\,a_x$

$400\cos 30 - \mu_k N = 50\,a_x$

$\Sigma F_y = m\,a_y$

$P\sin 30 + N - W = 50\,\cancel{a_y}^{0}$

$N = W - P\sin 30$

$N = (50)(9.81) + 200 = \boxed{290.5N}$

$\boxed{a_x = 5.185\ m/s^2}$

$V_i = 0 \qquad t_1 = 0$

$V_f = ? \qquad t_2 = 3$

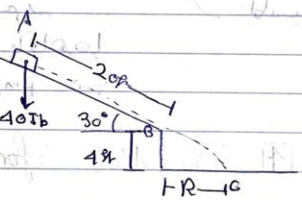
ادا القوى ثابتة مع
الزمن السارع ثابت
ادا كانت إحدى القوى
متغيرة يصبح التسارع
متغير

$V_f = V_a + at$

$V_f = (5.185)(3)$

$\boxed{V_f = 15.6\ m/s}$

13-18

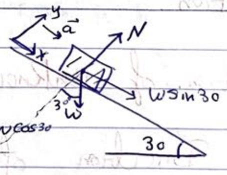


$W = 40\ Ib$

$V_A = 0$

Frictionless → Smooth

$R = ?$ and $L_{AC} = ?$



From A → B

$\Sigma F_x = m a_x$

$W\sin 30 = m a_x$

$\cancel{40}\sin 30 = \frac{\cancel{40}}{32.2}\, a_x$

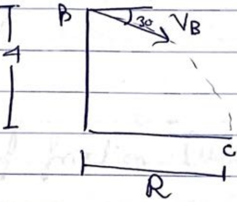
$\boxed{16.1 = a_x}$

③

$V_B^2 = V_A^2 + 2a_x (S_B - S_A)$

$V_B^2 = 0 + (2)(16.1)(20)$

$\boxed{V_B = 25.38 \text{ ft/s}}$



$V_B = V_A + a t_{AB}$

$25.38 = 16.1 \, t_{AB}$

$\boxed{t_{AB} = 1.576_s}$

$ys_C = ys_B + V_{By} \, t + \frac{1}{2} a t^2$

$-4 = 0 + {}^{-}(25.38)(\sin 30) t_{BC} + -\frac{1}{2}(32.2) t_{BC}^2$

$\boxed{t_{BC} = 0.2413 s}$

$x_C = x_B + V_{Bx} t_{BC}$

$R = 0 + (25.38)(\cos 30) \; t_{BC}$

$\boxed{R = 5.3 \text{ ft}}$

$t_{AC} = 0.2413 + 1.576 = 1.82 s$

① lec8

[1] Friction Force

Static Force
(at the avrege of Motion
$f_s = \mu_s N$

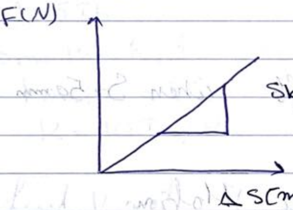
Kinematic Friction
$f_K = \mu_K N$

Where μ_s, μ_k : coefficients of friction (unitless)
$\mu_s > \mu_k$

$0 < \mu_s , \mu_k < 1$

for paeticls friction is always in the opposite Direction of Motion

[2] Spring Motion:



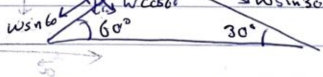
$F_s \propto \Delta S$

Slope = k $\quad F_s = k \Delta S$ [N] [M]

↳ coefficient of friction

B-17

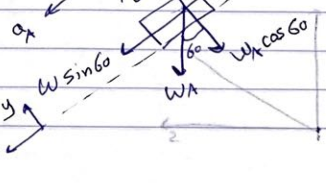
$W_A = W_B = 10 Ib$
$\mu_k = 0.1$



Paeticls / Kinetics / Rectilineare Motion

For A:-

$\Sigma F_x = W_A \sin 60 - T - \mu_K N_A$
$= m_A \, a_{Ax}$



$\Sigma F_y = N_A - W_A \cos 60 = 0$
$N_A = 5 Ib$

$\Sigma F_x = T - W_B \sin 30 - \mu_k N_B$

$= m_B a_{By}$

$\Sigma F_y = N_B - W_B \cos 30 = 0$

$N_B = 8.66$ lb

$L = S_A + S_B$ $\quad |a_A| = |a_B|$

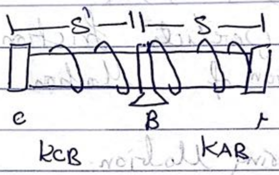
$0 = V_A + V_B$

$0 = a_A + a_B$

13-26

$K_{AB} = 2$ kN/m $\quad m = 2$ kg

$K_{BC} = $ ~~8~~ 3 kN/m

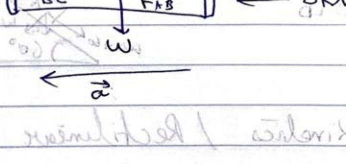


Unstretched = $S = S' = 250$ mm

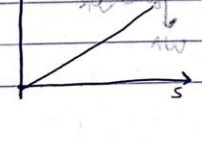
* Find the Speed of the Shaft when $S = 50$ mm

$S' = 450$ mm

Particle / Kinetics / Rectilinear Motion



$\Sigma F_x = 5000 - F_{AB} - F_{BC} = 2a$



Fs Change with position

→ changes with time

→ integration

$F_{AB} = K_{AB} \Delta S$
$= K_{AB} X = 2000 S$

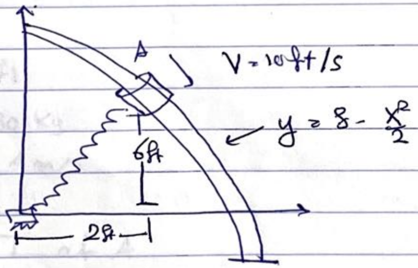
$F_{BC} = K_{BC} \Delta S = K_{BC} X = 3000 S$

$5000 - 2000X - 3000X = 2a$
$a = 2500 - 2500X$

$a\,d_x = V\,dv$
$\int_0^x 2500 - 2500x\,d_x = \int_0^v V\,dv$
$2500x - \frac{2500x^2}{2} = \frac{V^2}{2}$
$X = \Delta S = L_{st} - L_{unst} = 450 - 250 = 200mm$
So $V = 30$ m/s

B-84:-
W = 5 Ib
Smooth Rod $V_A = 10$ ft/s
l unst = 3 ft
K = 10 Ib/ft

find V and $|\vec{a}|$ at point A



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Exam Questio

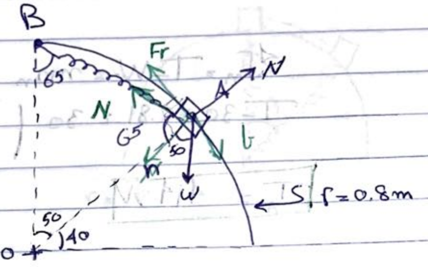
$m = 0.4$ kg

Unstretched length $= 0.4$ m

$k = 18$ N/m

$V_A = 2.4$ m/s

$\mu_k = 0.1$



Find $|\vec{a}_A|$. N

① Using Rule:-

$$\frac{OA}{\sin 65} = \frac{AB}{\sin 50}$$

$AB = 0.67$ m

$l_{st} = 0.67$

② $\boxed{\Sigma F_t = W \sin 50 - Fr - F_s \sin 65} = m a_t$

$(0.4)(9.81) \sin 50 - 0.1N - (18)(0.67 - 0.4) \sin 65 = 0.4\, a_t$

$\Sigma F_N = W \cos 50 + F_s \cos 65 - N = m a_n$

$(0.4)(9.81) \cos 50 + (18)(0.67 - 0.4) - N = 0.4 \boxed{a_n} \rightsquigarrow \frac{V^2}{r}$

③ Get N and Then find a_t

Then $a = \sqrt{a_t^2 + a_n^2}$

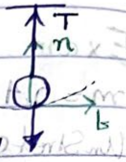
13-71

$m = 30$ kg

$V_A = 4$ m/s

① Find T at A

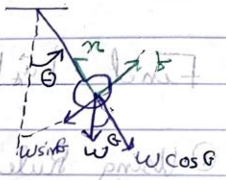
② θ to which the ball swings and momentarily stops.



$$\Sigma F_n = T - W = m a_n \leftarrow (4)^2$$

$$T - 30 \times 9.81 = 30 \left(\frac{V_A^2}{\rho} \right) \leftarrow 4$$

$$\boxed{T_A = 414\ N}$$



$$\Sigma F_t = -W \sin\theta = m a_t$$

θ متغيرة مع $\Rightarrow \boxed{a_t = -(9.81) \sin\theta}$

دايماً

متغير مع الزمن

$$\int a_t\, ds = V\, dv$$

$$S = \theta r$$

~~$ds = \theta dr$~~

$$ds = r\, d\theta$$

$$\int_0^\theta a_t\, r\, d\theta = \int_4^V V\, dv$$

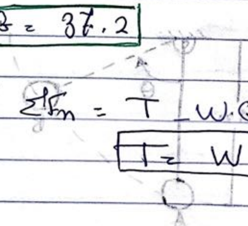
$$(-9.81)(4)(-\cos\theta]_0^\theta = \frac{V^2}{2} - \frac{(4)^2}{2}$$

$$(4)(9.81)[\cos\theta - 1] = \cancel{\frac{V^2}{2}}^{\nearrow 0} - 8$$

$$\cos\theta - 1 = \frac{-\cancel{8}2}{(\cancel{4})(9.81)}$$

$$\cos\theta = \left(\frac{-2}{9.81}\right) + 1$$

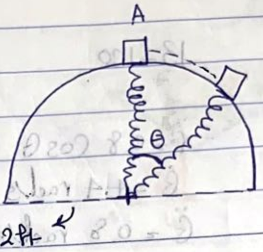
$$\boxed{\theta = 37.2}$$



$$\Sigma F_n = T - W \cdot \cos\theta = m \left(\cancel{a_n} \right)^{\cancel{V^2/\rho} \to 0}$$

$$\boxed{T = W \cos\theta}$$

Exam Question :-



$W = 2\ Ib$

$V_A = 0$

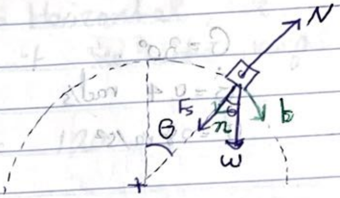
$k = 2 lb/ft$

$l_{uns} = ?$

$\theta = 60° \longrightarrow$ * رح نفلت عن السطح

ملاحظة: كل ما كبرت الزاوية قل ال cos

$N = 0$ at $\theta = 60°$



$$\Sigma F_t = W \sin\theta = m a_t$$

$\boxed{a_t = g \sin\theta}$ • تتغير مع الزمن

$$\Sigma F_n = W\cos\theta + F_s - N = m \frac{v^2}{\rho}$$

at $\theta = 60$

$$(2)\cos 60 + (\cancel{2})(F_s) - 0 = \left(\frac{2}{32.2}\right)\frac{v_B^2}{(2)}$$

To find v_B :-

$$\int a_t\, ds = \int_0^v v\, dv$$

$$\int_0^\theta g \sin\theta\, r\, d\theta = \frac{v^2}{2}$$

$$\frac{v^2}{2} = -gr\cos\theta + gr\cos 0$$

$$\frac{v^2}{2} = (1-\cos\theta)\, gr$$

$$v^2 = \left(1-\frac{1}{2}\right)\overset{32.2}{(\cancel{9.81})}(2) \times 2$$

$$v^2 = 2 \times 32.2$$

$v = \sqrt{}$

So F_s is known

$$F_s = k(\overset{2ft}{l_{st}} - l_{un})$$

$l_{un} = 1.5\ ft$

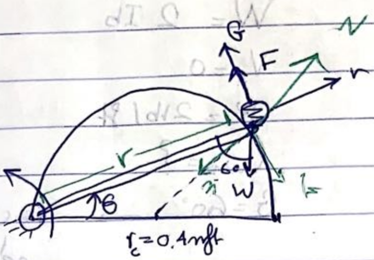
13-100

$r = 0.8 \cos\theta$ W = 0.5 lb

$\dot{\theta} = 0.4$ rad/s

$\ddot{\theta} = 0.8$ rad/s²

- Find The force from the arm on the ball when $\theta = 30°$ (F)



$\theta = 30°$

$\dot{\theta} = 0.4$ rad/s

$\ddot{\theta} = 0.8$ ra/s²

$r = 0.8 \cos(30) = 0.6928$ ft

$\dot{r} = -0.8 \sin\theta \; \dot{\theta}$

$\dot{r} = (-0.8)(\sin 30)(0.4)$

$\boxed{\dot{r} = -0.16 \text{ ft/s}}$

$\ddot{r} = (-0.8 \cos\theta)\dot{\theta}(\dot{\theta}) + \ddot{\theta}(-0.8)(\sin\theta)$

$\boxed{\ddot{r} = -0.431 \text{ ft/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2 \longrightarrow \boxed{a_r = -0.5417 \text{ ft/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \boxed{0.4263 \text{ ft/s}^2 = a_\theta}$

So $\boxed{a =}$

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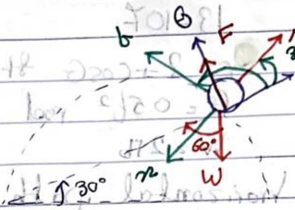
13-100 Continued



$a_r = -0.5417 \ ft/s^2$

$a_\theta = 0.1263 \ ft/s^2$

vertical path

We can find ψ using:-

Note: if the path is Horizontal, we can't see the weight.

$\tan \psi = r / \frac{dr}{d\theta} \Big|_{\theta = \text{desired}} = \frac{0.6928}{-0.4} = 120$

$r = 0.8 \cos \theta$

$r(30) = 0.8 \cos 30 = 0.6928 \ ft$

$\frac{dr}{d\theta} = 0.8 \sin \theta$

$\frac{dr}{d\theta}\Big|_{\theta=30} = -0.8 \ \sin 30 = -0.4$

So $\boxed{\psi = 120°}$

$\Sigma F_r = N \cos \theta - W \cos 60 = m a_r$

$= N \frac{\sqrt{3}}{2} - \frac{0.5}{2} = \frac{0.5}{32.2}(-0.5417)$

$\boxed{N = 0.279}$

$\Sigma F_\theta = F + N \sin 30 - W \sin 60 = m a_\theta$

$= F + (0.279)(0.5) - 0.5 \frac{\sqrt{3}}{2} = \frac{0.5}{32.2} (a_\theta) \leftarrow 0.1263$

$\boxed{F = 0.3 \ Ib}$

13-107

$r = 2 + \cos G$ ft

$\theta = 0.5t^2$ rad

$W = 2$ Ib

horizontal path

Find F_{rod} at $t = 1$ s

$G = 0.5t^2$

at $t = 1$

$\boxed{G = 0.5 \text{ rad}}$

$\dot{G} = t$

at $t = 1$

$\boxed{\dot{G} = 1 \text{ rad}}$

$r = 2 + \cos G$

$= 2 + \cos(0.5)$

$\boxed{r = 2.8776 \text{ ft}}$

$\ddot{G} = 1$

at $t = 1$

$\boxed{\ddot{G} = 1 \text{ rad}}$

$\dot{r} = -\sin\theta \; \dot{\theta}$

$\dot{r} = (-\sin 0.5)(1)$

$\boxed{\dot{r} = -0.4974 \text{ ft/s}}$

$\ddot{r} = (-\sin\theta)\ddot{\theta} + \dot{\theta}(-\cos G)\dot{\theta}$

$\boxed{\ddot{r} = -1.357 \text{ ft/s}^2}$

So $\boxed{\begin{array}{l} a_r = -4.2346 \text{ ft/s}^2 \\ a_\theta = 1.9187 \text{ ft/s}^2 \end{array}}$

$\psi = \tan^{-1} r \Big/ \frac{dr}{dG}$ at $G = 0.5$ $\boxed{\text{rad}}$

$= \tan^{-1} 2.8776 \Big/ -\sin G \leftarrow 0.5$

$\boxed{\psi = 99.46^\circ}$

$\alpha = 99.46 - 90. = 9.46^\circ$

$\sum F_r = N \cos 9.46 = m a_r$

$N \cos 9.46 = \frac{2}{32.2} (-4.2346)$

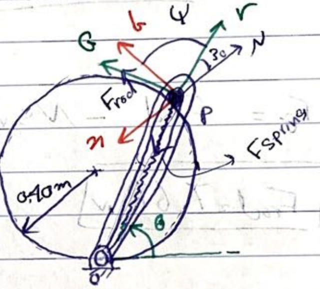
$N = -0.2666$ Ib

$\sum F_\theta = N \sin \cos\alpha + F_{rod} = m a_\theta$

$= F_{rod} - 0.2666 \sin 9.46 = \frac{2}{32.2} (1.9187)$

$\boxed{F_{rod} = 0.163 \text{ Ib}}$

13-109



$OP = r = 0.8 \sin\theta$

$m = 80$ g

horizontal path

$k = 30$ N/m

$l_{un} = 0.25$ m

$\theta = 60°$ $\dot{\theta} = 5$ rad/s Constant

Find Frod

$a_r = -34.641$ m/s²

$a_\theta = 20$ m/s²

$r = 0.8 \sin\theta$

$r = 0.6928$ at $\theta = 60$

$\dot{r} = 0.8 \cos\theta \, \dot{\theta}$

$\dot{r} = 2$ m/s

$\ddot{r} = (0.8) \left(\overset{0}{\cancel{\cos\theta \, \ddot{\theta}}} + - \dot{\theta} \sin\theta \, \dot{\theta} \right)$

$\ddot{r} = -17.321$ m/s² at $\theta = 60$

$$\psi = \tan^{-1} r \Big/ \frac{dr}{d\theta} \Big|_{\theta = 60^\circ}$$

$$= \tan^{-1} \frac{0.6928}{0.8 \cos 60} =$$

$$\boxed{\psi = 60^\circ}$$

$$\sum^{\ell} F_r = + N\cos 30 - F_s = m a_r$$

$$N\left(\frac{\sqrt{3}}{2}\right) - K\left(\underset{\overset{\nearrow}{0.6928}}{l_{st}} - \underset{\overset{\nearrow}{0.25}}{l_{un}}\right) = 80\times10^{-3}(-34.641)$$

So $\boxed{N = 12.1\ N}$

$$\sum^{\prime} F_\theta = F_{rod} - N\sin 30 = (80\times10^{-3})(20)$$

$$\boxed{F_{rod} = 7.67\ N}$$

Chapter 13 is finished