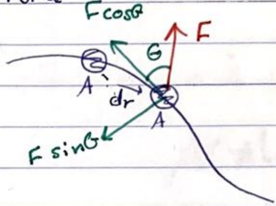
Chapter 14:- Kinetics of a Particle (Work & Energy)

• 14.1 Work of a Force

A Force does a Work on a particle only when the particle UnderGoes a displacement in the direction of the Force



$dU = |\vec{F}| \cos\theta \, |\vec{dr}|$

$\boxed{dU = \vec{F} \cdot \vec{dr}}$

→ Scalar Quantity

$dU = \vec{F} \cdot \vec{dr}$
$= N.m = [\text{Joule}]$
$= [lb \, . \, ft]$ (No Name)

$dU = |F| \, |dr| \cos\theta$

- If $0 \leq \theta < 90$ → $\cos\theta$ is +ve → dU +ve
  (Force and displacement have the same sense)
- If $\theta = 90$ → $\cos\theta = 0$ → No Work is done
- If $90 < \theta \leq 180$ → $\cos\theta$ is −ve → du −ve
  (Force and displacement have the opposite Sense)

$$U_{1-2} = \int_{r_1}^{r_2} |\vec{F}| \cos\theta \, |\vec{dr}|$$

- U is the Area Under the Curve

$$U_{1-2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

1] Variable Force

$\vec{F}(s)$: Force Changes with position

$$U_{1-2} = \int_{s_1}^{s_2} F(s) \cos\theta \, ds$$

2] Constant Force

$$U_{1-2} = F \cos\theta \, (s_2 - s_1)$$

$N \neq W$

3] Weight

$$U_{W_{1-2}} = \int_{r_1}^{r_2} -W\hat{j} \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= -\int_{y_1}^{y_2} W \, dy$$

$$U_{W_{1-2}} = -W(y_2 - y_1) = -W\Delta y$$

4] Spring

$F_s = (-)ks$ ضد ثابت الـ spring إزاحة الجسم

$$U_{s_{1-2}} = \int_{s_1}^{s_2} -ks \, ds \Rightarrow U_{s_{1-2}} = -\frac{1}{2}k(s_2^2 - s_1^2)$$

$s_1 = l_{st_1} - l_{unst}$

$s_2 = l_{st_2} - l_{unst}$

5) Friction

$$U_{Fr_{1-2}} = -\mu_k N_{\#} s'$$

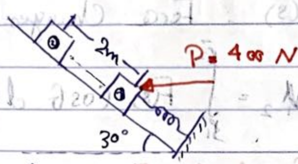
***Note:-**

All reactive forces do No work at all times
– (Normal / Tension / pins / rollers ....)

Ex 14.1

$k = 30\ N/m$

$m = 10\ kg$



Spring was initially stretched 0.5m

Find $U_{T_{1-2}}$

① $U_{F_s\,1-2} = -\frac{1}{2} k\,(S_2^2 - S_1^2)$

$= -\frac{1}{2}(30)\,(2.5^2 - 0.5^2)$

$U_{F_s\,1-2} = -96$ ‏. عكسها الى لانه القوه ضد الحركة

② $U_{P_{1-2}} = (400)\cos\theta\,\Delta S$

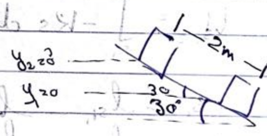
$= (400)(\cos 30)(2)$

$U_{P_{1-2}} = 692.8$ ‏. الشغل موجب

③ $U_{W_{1-2}} = -W(y_2 - y_1)$

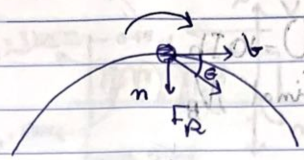
$= -(10)(9.81)(2\sin 30 - 0)$

$U_{W_{1-2}} = -98.1\ J$



$U_T = 692.8 - 90 - 98.1 = 505\ J$

14.2 / 14.3 :- Principle of Work and Energy



$$\Sigma F_t = F_R \cos\theta = m a_t$$
$$\Sigma F_n = F_R \sin\theta = m \frac{v^2}{\rho}$$

$$a_t \frac{ds}{ds} = \frac{v\,dv}{ds}$$

$$a_t = v \frac{dv}{ds}$$

$$F_R \cos\theta = m v \frac{dv}{ds}$$
$$F_R \cos\theta \, ds = m v \, dv$$

$$\int_{s_1}^{s_2} F_R \cos\theta \, ds = \int_{v_1}^{v_2} m v \, dv$$

$$U_{T_{1-2}} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

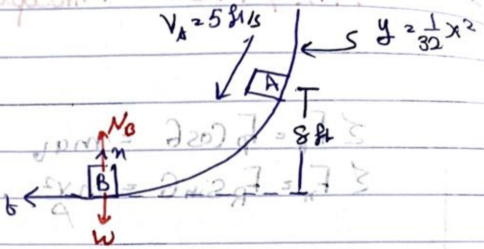
$$\boxed{\frac{1}{2} m v_1^2 + U_{T_{1-2}} = \frac{1}{2} m v_2^2}$$

Kinetic Energy

$$T_1 + U_{1-2} = T_2$$ → All variables here are Scalars

← بستعملوها كل المعطيات تكون: Forces/ positions/ velocities

14-19



$\rightarrow$ $W = 10$ lb

Find N_B

$\Sigma F_n = N_B - W = m a_{Bn}$??

$N_B - 10 = \frac{10}{32.2} \frac{V_B^2}{\rho_B}$

$U_W = -W(y_B - y_A)$

$= -10(0-8) = 80$ lb. ft

$T_A + U_{A-B} = T_B$

$(\frac{1}{2})(\frac{10}{32.2})(5)^2 + 80 = \frac{1}{2}(\frac{10}{32.2})V_B^2$

$V_B = 23.24$ ft/s

$y_B = 0 \rightarrow x_B = 0$

$\rho_B = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ => $\frac{dy}{dx} = \frac{2}{32}x = 0$

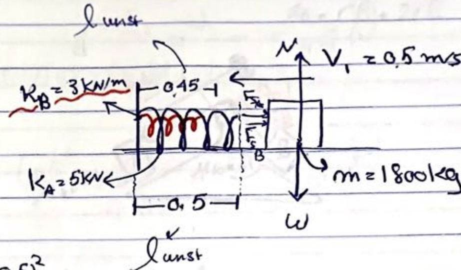
$\frac{d^2y}{dx^2} = \frac{2}{32}$

So $\rho_B = 16$ ft

$N_B = 20.5$ lb

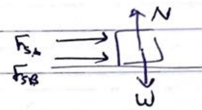
14-20

• Determine the Maximum deflection



$T_1 = \frac{1}{2} m V_1^2 = \frac{1}{2}(1800)(0,5)^2$
$= 225\ J$

$T_2 = 0$



$T_1 + U_{1-2} = T_2$

$225 + U_{1-2} = 0$

$U_{1-2} = -225\ J$

$F_{SA} + F_{SB} = ma$

$U_T = U_{F_{SA}} + U_{F_{SB}}$

Note $S_{A_2} = l_{s_{A_2}} - l_{uns}$

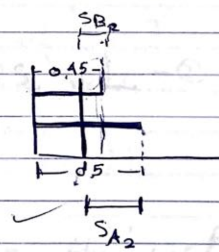
$S_{A_1} = l_{s_{A_1}} - l_{unst}$
$= 0,5 - 0,5 = 0$

$= -\frac{1}{2} K_A (S_{A_2}^2 - S_{A_1}^2) + -\frac{1}{2} K_B (S_{B_2}^2 - S_{B_1}^2)$

$= -\frac{1}{2} 5000 (S_{A_2}^2 - 0) + -\frac{1}{2}(3000)(S_{B_2}^2 - 0)$

$\cancel{-}225 = \cancel{-}2500\, S_{A_2}^2 \cancel{-} 1500\, S_{B_2}^2$ — ①

$225 = 2500\, S_{A_2}^2 + 1500\, S_{B_2}^2$



$S_{A_2} = S_{B_2} + 0,5$

$225 = 2500 (S_{B_2} + 0,5)^2 + 1500\, S_{B_2}^2$

solving:-

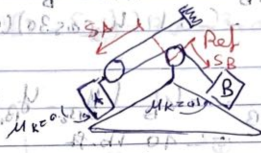
$S_{B_2} = 0,205m,\ S_{A_2} = 0,255\ m$

$a_r = \ddot{r} - r\dot{\theta}^2$ lec12
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

14-16.

$W_A = 60$ Ib

$W_B = 40$ Ib

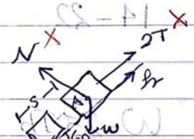


Both start from rest.

Block B moves 2 ft up the incline.
Find V_A at that instant

For Several particles:-

$$\sum T_1 + \sum U_{1-2} = \sum T_2$$
$$\sum T_1 = T_{1A} + T_{1B} = 0 \text{ Ib.ft}$$
$$\sum T_2 = T_{2A} + T_{2B}$$
$$= \frac{1}{2} m_A V_{A_2}^2 + \frac{1}{2} m_B V_{B_2}^2$$
$$= (\tfrac{1}{2})\left(\frac{60}{32.2}\right) V_{A_2}^2 + \frac{1}{2}\left(\frac{40}{32.2}\right) V_{B_2}^2$$



T, N doesn't do any Work

For A :-

$$U_{fr_A} = -\mu_K N S_A = (-0.1)(W_A \cos 60) S_A$$

$$U_{W_A} = -W_A(y_{A_2} - y_{A_1}) = (-60)(0 - S_A \sin 60) = 60 \sin 60 S_A$$

$2S_{A_1} + S_{B_1} = L$ —① and $2S_{A_2} + 2S_{B_2} = L$ —②

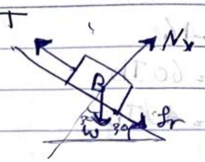
~~$2V_A + V_B = 0$~~

① - ② : $2\Delta S_A + \Delta S_B = 0$

A moved 1 ft down the incline

So $U_{fr_A} =$

$$U_{fr_B} = -\mu_k N S_B$$
$$= (-0.1)(W_B \cos 30)(2)$$



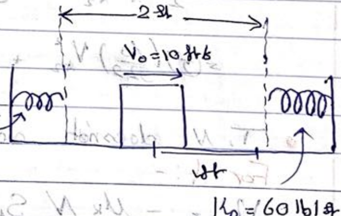
$$U_{W_B} = -W_B (y_{B_2} - y_{B_1})$$
$$= -40 \text{ lb.ft}$$

$$\sum T_1 + U_{fr_A} + U_{W_A} + U_{fr_B} + U_{W_B} = \left(\tfrac{1}{2}\right)\left(\tfrac{60}{32.2}\right) V_{A_2}^2 + \left(\tfrac{1}{2}\right)\left(\tfrac{40}{32.2}\right) V_{B_2}^2$$

$$2V_A + V_B = 0$$

$V_A = 0.771$ ft/s
$V_B = -1.54$ ft/s

14-22.



$W = 25$ lb
$\mu_k = 0.4$
Find S_T when it comes to Rest.

$$T_0 = \tfrac{1}{2} m V_0^2 = \tfrac{1}{2}\left(\tfrac{25}{32.2}\right) 10^2 \quad \text{lb.f}$$
$T_1 = 0 \rightarrow$ لأن الجسم توقف كلياً

$$U_{fr} = (\mu_k)(N)\ S = (-0.4)(25)(1 + S_{B_2})$$

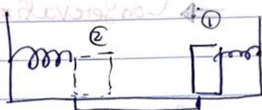
$$U_{sp} = \left(-\tfrac{1}{2}\right)(K_B)(S_{B_1}^2 - S_{B_2}^2)$$

$$= \left(-\tfrac{1}{2}\right)(60)(S_{B_1}^2 - 0) = -30 S_{B_1}^2$$

$$T_0 + U_{0-1} = T_1$$

$$(\tfrac{1}{2})(\tfrac{25}{32.2})(10)^2 + (-0.4)(25)(1 + S_{B_1}) - 30 S_{B_1}^2 = 0$$

$$S_{B_1} = 0.8275 \text{ ft}$$



$$T_1 = 0$$

$T_2 = 0$ لان الجسم المفروض يوقف حسب فرضنا له $2 + 0.8275 = 2.8275$

$$U_{fr} = -\mu_k N x$$
$$= (-0.4)(25)\, x$$
$$=$$

$$U_{sp} = -(\tfrac{1}{2}) k_B (S_{B_2}^2 - S_{B_1}^2)$$
$$= -\tfrac{1}{2}(60)(0 - (0.8275)^2)$$
$$= (30)(0.8275)^2$$

$$T_1 + U_{1-2} = T_2$$
$$0 - (0.4)(25)(x) + 30(0.8275)^2 = 0$$
$$x = 2.05 \text{ ft}$$

← لو طلع الـ x اكبر من 2.8275

$U_{fr} = (-0.4)(25)(2.8275 + S_{A_2})$ نحل

$$U_{sp_A} = -\tfrac{1}{2} k_A S_{A_2}^2$$

$$U_{sp_B} = -\tfrac{1}{2} k_B (0 - (0.8275)^2)$$

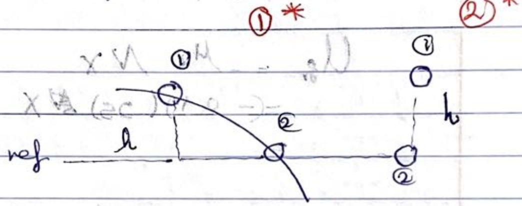
$$S_T = 1 + 0.8275 + 2.05 = 3.88 \text{ ft}$$

14.4 is not included (Power, efficien.

14.5/14.6 : Conservative Forces / Potential Energy
⟹ Conservation of Energy

Def: Conservative Force : It's work does not depend on the path that the particle takes.

Examples:- 1) Weight



$U_w = -w(y_2 - y_1)$
$= -w(0 - h)$

$U_w = wh$ for ①*

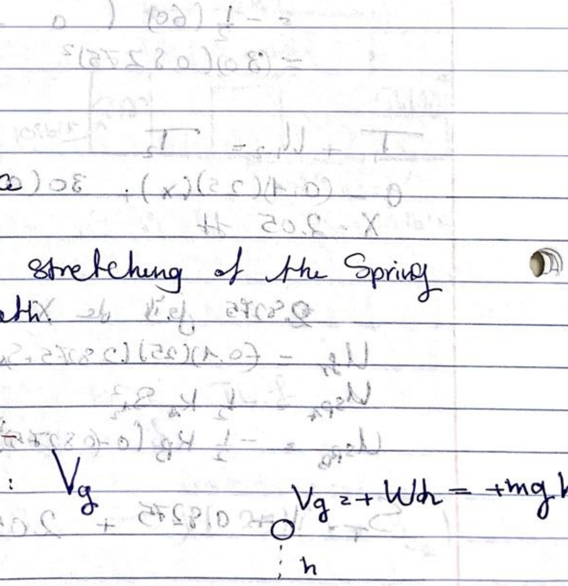
$U_w = wh$ for ②*

2) Spring Force

U_{sp} depends on the stretching of the Spring and not the path.

Potential Energy

1. Gravitational P.E : V_g



2 Elastic P.E (always Positive)

$V_e = \frac{1}{2} K S^2$ دايماً تكبس الـ spring طاقة

$S: l_{st} - l_{unst}$

$V_T = V_g + V_e$

- If only Conservative forces are doing work on a particle, the energy is conserved.

$$T_1 + V_1 = T_2 + V_2$$

* Note:

$F_s = k s$

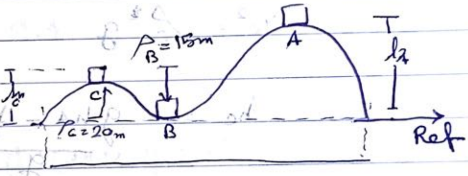
$U_{s_{1-2}} = -\frac{1}{2} k (s_2^2 - s_1^2)$

$V_s = +\frac{1}{2} k s^2$ $\qquad s = l_{st} - l_{unst}$

14-37:-

$N \neq 0$ at C

$N < 4\ mg$ at B



Find h_A, h_C?

→ No friction

$\boxed{V_A = 0}$

$A \longrightarrow B$

$$\cancel{T_A}^{0} + V_A = \cancel{T_B} + \cancel{V_B}$$

$$(\cancel{m}g)(h_A) = \frac{1}{2}\cancel{m_B} V_B^2 + 0$$

$$g\ h_A = \frac{V_B^2}{2}$$

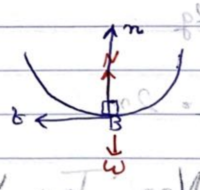
$$2g\ h_A = V_B^2$$

$$h_A = \frac{V_B^2}{2g} \sim ?$$

So $h_A = \frac{3g\ \rho_B}{2g}$

$= \left(\frac{3}{2}\right)(15)$

$= \frac{45}{2} = 22.5\ m$



$$\sum F_n = N - W = m\ a_n$$

$$4W - W = m\ a_n$$

$$4mg - mg = m\ a_n$$

$$3g = a_n$$

$$3g = \frac{V^2}{\rho_B}$$

$$V_B^2 = 3g\ \rho_B$$

$T_A + V_A = T_C + V_C$

$0 + mgh_A = \frac{1}{2} m V_C^2 + mgh_C$

$g h_A = \frac{1}{2} V_C^2 + g h_C$

we need V_C?

$-N + W = m a_n$

$0 + W = m \frac{V_C^2}{\rho_C}$

$\cancel{m} g = \cancel{m} \frac{V_C^2}{\rho_C}$

$V_C^2 = \rho_C g$

So $h_C = \frac{g h_A - \frac{1}{2} V_C^2}{g}$

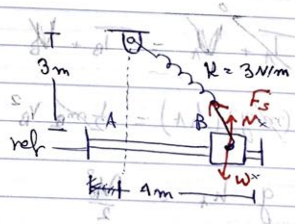
$h_C = (9.81)(22.5) - (\frac{1}{2})(20)(9.81) / 9.81$

$h_C = 22.5 - 10 = 12.5$ m

→ h_C should be done higher

14-73

$m = 2$ Kg



→ $l_{unst} = 2$ m

$T_B + V_B = T_A + V_A$

$0 + (\frac{1}{2})(K) S_B^2 = \frac{1}{2} m V_A^2 + \frac{1}{2} K S_A^2$

$K S_B^2 = m V_A^2 + K S_A^2$

$K(3)^2 = m V_A^2 + K(1)^2 \quad = \quad 9K - K = m V_A^2$

$V_A = 3.46$ m/s^2

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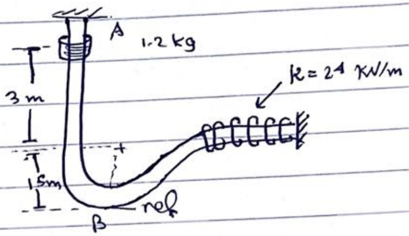
$V_A = 0$

Find max deflection of the Spring.



$$\cancel{T_A}^{0} + V_A = T_B + V_B$$

$$V_A = T_B + V_B$$

$$mg(4.5) = T_B + 0$$

$$\cancel{m}g\ (4.5) = \frac{1}{2}\cancel{m}\ V_B^2$$

$$V_B = \sqrt{(2g)(4.5)} = 9.396 = 9.4\ m/s^2$$

$$T_A + V_A = T_C + V_C$$

V_{ce} V_{cg}

$$0 + (mg)(4.5) = 0 + \left((mg)(1.5) + \left(\frac{1}{2}\right)(24\,K)(S)^2 \right)$$

$$m\,g\,(4.5) = mg(1.5) + \left(\frac{1}{2}\right)(24\,K)\,(S)^2$$

$(S)' =$