Chapter 15: Kinetics of a particle: Principle of linear Impulse & Momentum.

$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{F} = m\frac{dv}{dt}$$

$$\int_{t_1}^{t_2} \sum \vec{F}\, dt = \int_{v_1}^{v_2} m\, dv$$

$$\int_{t_1}^{t_2} \sum \vec{F}\, dt = m(\vec{V}_2 - \vec{V}_1)$$

$$m\vec{V}_1 + \int_{t_1}^{t_2} \sum \vec{F}\, dt = m\vec{V}_2$$ → Principle of linear impulse & Momentum

- **linear Momentum :-**

$$\vec{L} = m\vec{V}$$

→ same direction as the velocity ⇒ [Kg.m/s] [slug.ft/s] (Units)

- **linear impulse :-**

$$\vec{I} = \int_{t_1}^{t_2} \sum \vec{F}\, dt$$ same direction as $\sum \vec{F}$ ⇒ [N.s] [lb.s] (Units)

- **Note :-** For a Group of particles

$$\sum (m_i v_i)_1 + \sum \int_{t_1}^{t_2} \sum (F)\, dt = \sum (m_i v_i)_2$$

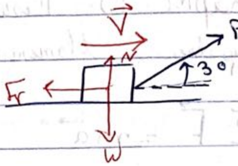
15-10:

m = 50 kg

$V_1 = 0$ m/s
$V_2 = 10$ m/s } 5 s

$\mu_k = 0.2$

Find P



$N + P\sin 30 = W$

$$m V_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = m V_{x_2}$$

$$(50)(0) + \int_0^5 \Sigma F_x \, dt = (50)(10)$$

$$\int_0^5 (P\cos 30 - \mu_k N) \, dt = 500$$

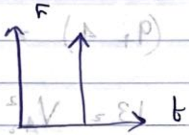
$$P\cos 30 (5) - (0.2)(50)(9.81)(5) = 500$$
$$- P\sin 30)$$

So P = 20.5 N
N = 387.97 N

15.3

Conservation of linear Momentum :-

$$\int_{t_1}^{t_2} \sum \vec{F} \, dt =$$



$\sum \vec{F} \uparrow\uparrow \simeq \infty$

$\Delta t \downarrow\downarrow \simeq 0$

$$\int_{t_1}^{t_2} \sum \vec{F} \, dt = 0$$

impulsive forces

$$\sum m_i v_{i_1} = \sum m_i v_{i_2}$$

Example :- 15.4

(1) $\xrightarrow{1.5 \, m/s}$ (A) $\xleftarrow{0.75 \, m/s}$ (B)

$m_A = 15 \, Mg$, $m_B = 12 \, Mg$ } collision + Coupling. Find V_2

$$m_A \vec{V}_{A_1} + m_B \vec{V}_{B_1} = (m_A + m_B) \vec{V_2}$$

$$(15 \times 10^3)(1.5) - (0.75)(12 \times 10^3) = (27 \times 10^3)(V_2)$$

$V_2 = 0.5 \, m/s \rightarrow$

(2) 15.6

$\xrightarrow{3 m/s}$ (A) $\xleftarrow{2 m/s}$ (B)

$m_A = m_B = 150 \, kg$

Find V_{B_2}, V_{A_2} (No energy was lost during impact)

$$m_A V_{A_1} + m_B V_{B_1} = m_A V_{A_2} + m_B V_{B_2}$$

$$(3) - (2) = V_{A_2} + V_{B_2}$$

$$V_{A_2} + V_{B_2} = 1$$

$$\frac{1}{2} m_A V_{A_1}^2 + \frac{1}{2} m_B V_{B_1}^2 = \frac{1}{2} m_A V_{A_2}^2 + \frac{1}{2} m_B V_{B_2}^2$$

$$(9 + 4) = V_{A_2}^2 + V_{B_2}^2$$

$$13 = V_{A_2}^2 + V_{B_2}^2$$

$V_{A_2} = -2$

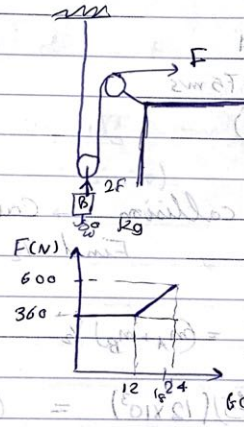
$V_{B_2} = +3$ or -2 but the right one is $+3$

Example: 15-28

$t = 24$ s

$V_{B_1} = 0$

Find V_2 at $t = 18$ s



$$m_1 v_1^{\,0} + \int_{t_1}^{t_2} \sum F_y \, dt = m_2 v_2$$

$$\int_0^{18} (2F - W)\, dt = 50 V_2$$

$y = ax + b$ ← y-intercept, a ← slope

$360 = 240 + b$

$120 = b$

$$a = \frac{600 - 360}{12} = 20$$

$$(20) = \frac{360 - b}{12 - 0} \rightarrow b = 120 \quad \text{So } F(t) = 20t + 120$$

lec 15

1

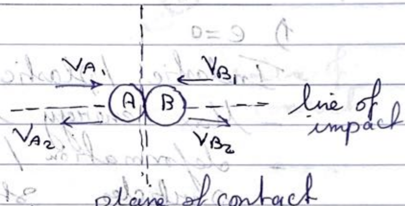
Impact:-

- 2 or more Bodies
- collision
- high impulsive Forces } → Conservation of linear momentum
- short period of time }

There are two types of impact

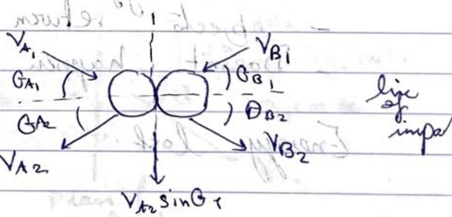
1) Central impact:-

$m_A \vec{V}_{A_1} + m_B \vec{V}_{B_1} = m_A \vec{V}_{A_2} + m_B \vec{V}_{B_2}$



{• Directions of motion along the line of impact}

2) Oblique impact



① $m_A(V_{A_1}\cos\theta_{A_1}) + m_B(V_{B_1}\cos\theta_{B_1}) = m_A(V_{A_2}\cos\theta_{A_2}) + m_B(V_{B_2}\cos\theta_{B_2})$

② ~~$V_A \cos$~~ $\begin{cases} V_{A_1}\sin\theta_{A_1} = V_{A_2}\sin\theta_{A_2} \\ V_{B_1}\sin\theta_{B_1} = V_{B_2}\sin\theta_{B_2} \end{cases}$

{• Direction of motion makes an angle along the line of impact}

Lec. 5

1) Coefficient of Restitution:-

$$e = \frac{V_{B_2} - V_{A_2}}{V_{A_1} - V_{B_1}}$$ "along the line of impact"

↳ Depends on the material / shape / velocity

$0 \leq e \leq 1$

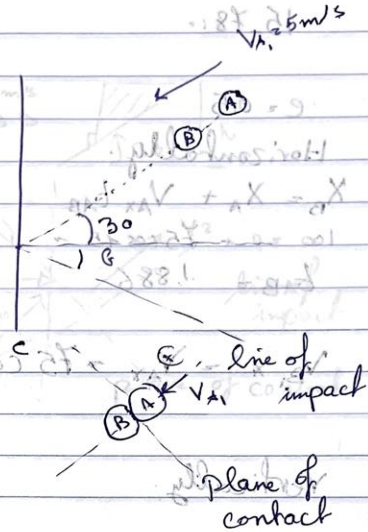
1) $e = 0$
Inelastic / plastic
- Max energy loss
- deformation / destruction
- particles stick together

2) $e = 1$
elastic impact (perfect)
- No energy loss
- objects return to their original state
- Doesn't happen in reality.

Energy lost: $T_{lost} = \sum T_2 - \sum T_1$ → negative

15-77 :-

$e_{AB} = 0.8$
$e_{BC} = 0.6$
$m_A = m_B = 0.4$



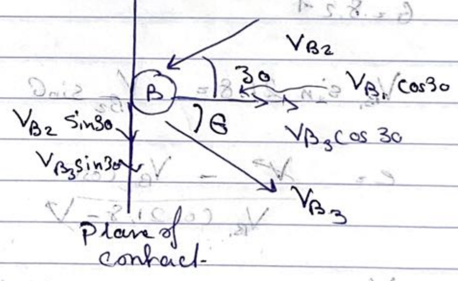
$$m_A V_{A_1} + m_B V_{B_1} = m_A V_{A_2} + m_B V_{B_2}$$

$$5 + 0 = V_{A_2} + V_{B_2}$$

$$e = \frac{V_{B_2} - V_{A_2}}{V_{B_1} - V_{B_1}}$$

$$0.8 = \frac{V_{B_2} - V_{A_2}}{5}$$

$V_{A_2} = +0.5$ m/s
$V_{B_2} = 4.5$ m/s



$$V_{B_2}\sin 30 = V_{B_3}\sin\theta$$

$$e = 0.6 = \frac{V_{C_3} - V_{B_3}\cos\theta}{V_{B_2}\cos 30 - V_{C_2}}$$

$V_{B_3} = 3.24$ m/s $\qquad \theta = 43.4°$

15_78 :>

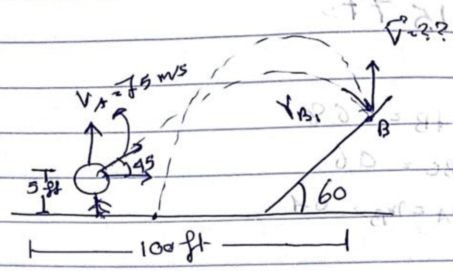
$e = 0.5$

Horizontally:

$X_B = X_A + V_{Ax}\, t_{AB}$

$100 = 0 + 75 \cos 45$

$t_{AB} = 1.886$



$V_{B_1 x} = V_{Ax} = 75 \cos 45 = 53.59$ ft/s

vertically:

$V_{B_1 y} = V_{Ay} - g\, t_{AB}$

$V_{B y} = -7.684$ ft/s

$|\vec{V}_{B_1}| = 53.03$

$\theta = 8.21$

$V_{B_1} \sin 21.8 = V_{B_2} \sin\theta$

$e = \dfrac{V - V_{B_2} \cos\theta}{V_{B_1} \cos 21.8 - V}$

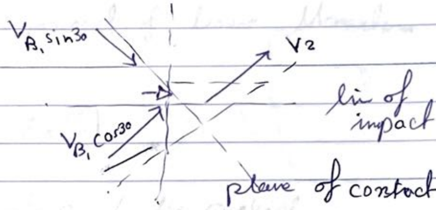
$V_{B_2} = 31.8$ $\theta = 14.4$

15.57

$m = 10\ Kg$

$m_0 = 10g$

$m v_1 + m_0 v_{0_1} = (m + m_0) v_2$



$\frac{10}{1000} (300) \cos 30 =$

$\left(10 + \frac{10}{1000}\right) V_2$

$V_2 = 0.2595$ m/s

$T_2 + V_2 = T_3 + V_3$

$\frac{1}{2}\left(10 + \frac{10}{1000}\right)(0.2595)^2 + mg(0) = 0 + mgh$

$= \left(10 + \frac{10}{1000}\right) 9.81 \times h$

$h = ??$

$d = \frac{h}{\sin 30} = 6.87\ m$

15.5 Angular Momentum

$\vec{L} = m\vec{v}$

Angular Momentum: the moment of linear Momentum about a point O

$\vec{H}_o = \vec{r} \times m\vec{v}$ $\quad (\vec{M}_o = \vec{r} \times \vec{F})$

$\vec{r}$: position vector from point O to the particle

$H_o = \vec{r} \times m\vec{v}$ $\quad [kg \cdot m^2/s]$ or $[slug \cdot ft^2/s]$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix} = i(r_y m v_z - r_z m v_y) - j(r_x m v_z - r_z m v_x) + k(r_x m v_y - m v_x r_y)$$

- Direction is according to right hand rule.

15.6: Relation between moment of a Force and the Angular Momentum.

$r \times \left(\sum \vec{F} = m\vec{a} = m\dot{\vec{v}}\right)$

$r \times \sum \vec{F} = r \times m\dot{\vec{v}}$

$\sum \vec{M}_o = \vec{r} \times m\dot{\vec{v}}$

$\vec{H}_o = \vec{r} \times m\vec{v}$

$\dot{\vec{H}}_o = \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}}$

$= \vec{v} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}}$

$\dot{\vec{H}}_o = \vec{r} \times m\dot{\vec{v}} = \sum \vec{M}_o$

$$\sum M_o = \dot{\vec{H}}_o$$

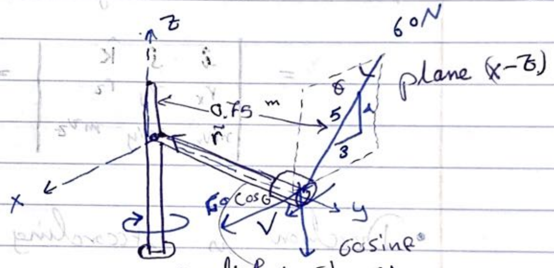
$$\sum \vec{M}_o = \frac{d\vec{H}_o}{dt}$$

$$\int_{t_1}^{t_2} \vec{M}_o \, dt = \vec{H}_{o_2} - \vec{H}_{o_1}$$

$$H_{o_1} + \int_{t_1}^{t_2} \sum \vec{M}_o \, dt = \vec{H}_{o_2}$$ → Principle of Angular impulse and Momentum

$$m\vec{V}_1 + \int_{t_1}^{t_2} \sum \vec{F} \, dt = m\vec{V}_2$$

15-100:-



Particle النقطة

So $r = 0.75\hat{j}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$M = 8t^2 + 5$ N.m

$m = 10$ kg

$V_1 = 2$ m/s at $t=0$

Find V_2 at $t = 2$ s

$$\vec{H}_{o_1} + \int_{t_1}^{t_2} \sum M \, dt = \vec{H}_{o_2}$$

$$\vec{H}_{o_1} = \vec{r} \times m\vec{V}_1 = 0.75\hat{j} \times 10\,(2\hat{i})$$

$$= 15 \quad -\hat{k} = -15\hat{k} \quad \text{kg m}^2/\text{s}$$

$$\vec{H}_{o_2} = \vec{r} \times m\vec{V}_2 = 0.75\hat{j} \times 10\,(V_2)\,\hat{i}$$

$$= -7.5 V_2 \hat{k} \quad \text{kg m}^2/\text{s}$$

$$\int_0^2 (8t^2 + 5)\, dt + (60)(\tfrac{3}{5})(0.75) = \left(\frac{8t^3}{3} + 5t\right) + \quad -\hat{k}$$

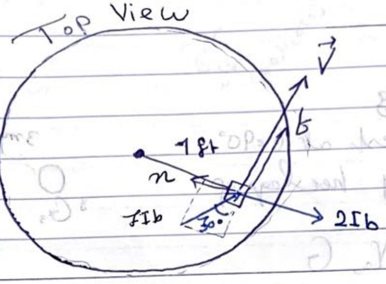
in $-\hat{k}$ $\quad (36)(\frac{75}{100})$

So: $15 + \int_0^2 8t^2 + 5 + 60\left(\frac{3}{5}\right)(0.75)\, dt = 7.5 V_2$

$\underline{V_2 = 13.4 \text{ m/s}}$

15 — 101 :-

$W = 10$ lb
$\mu_k = 0.5$
$V_1 = 2$ ft/s
Find t until
$T = 20$ lb

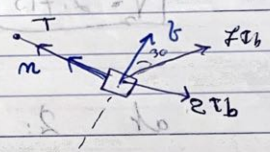


Angular Momentum

$$\vec{H}_{o_1} + \int_{t_1}^{t_2} \sum M_o \, dt = \vec{H}_{o_2}$$

$$\vec{H}_{o_1} = (4)\left(\frac{10}{32.2}\right)(2)\, k =$$

$$\vec{H}_{o_2} = (4)\left(\frac{10}{32.2}\right) V_2 =$$



$$T - 2 + 7\sin\theta = m a_n$$

$$20 - 2 - 7\sin\theta = \frac{10}{32.2} \times \frac{V_2^2}{4}$$

$\boxed{V_2^2 = 13.67}$

$$H_{o_1} + \int_0^t \sum M_o \, dt = H_{o_2}$$

$$H_{o_1} + \int_0^t (7\cos 30)(4)\, dt = H_{o_2}$$

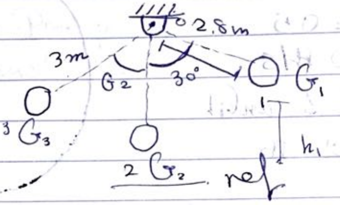
$t = 3.41$ s

15.7: Conservation of Angular Momentum

$\vec{H}_{o_1} = \vec{H}_{o_2}$ if $\Sigma \vec{M}_o = 0$

15-108

m = 50 kg
from rest at θ = 90°
holding her legs up
Find V_2, θ



Before letting her legs down.

$T_1 + V_1 = T_2 + V_2$

$(\frac{1}{2})(m)(0) + mg[(2.8) - (2.8)\cos 30] = (\frac{1}{2})(50)V^2 + 0$

$V_2 = 2.713$ (Just Before letting her legs down)

at 2: $\Sigma \vec{M}_o = 0$

So $\vec{H}_{o_2} = \vec{H}_{o_2}$

$r \times mV_{2\,before} = r \times mV_{2\,after}$

$(2.8) \times 50 \times 2.713 = (3)(50)\, V_{2\,after}$

so V_2 after = 2.53 m/s

$T_2 + V_2 = T_3 + V_3$

$(\frac{1}{2})(50)(2.53)^2 + 0 = 0 + (50)(9.81)(h_2)$

$h_2 =$ where $h_2 = 3 - 3\cos\theta$

so θ = 27°

$$\int_0^{12} \left(2(360) - 80(9.81)\right) dt + \int_{12}^{15} 2(20t + 120) - 80 \times 9.81 \, dt = 80v_2$$

$v_2 = 16.6$ m/s

$$\text{Area} = \int_0^{18} F \, dt$$

for the previous