Chapter 17: Kinetics of a Rigid Body

Force & Acceleration

17.1:

Force on Rigid Body → Translation + Rotation

$\sum F = ma$ ← قانون نيوتن العادي

$\sum \vec{M}_o = I_o \vec{\alpha}$

$\sum M_o$ → Moments about O

α → Ang. acc

I_o: Mass Moment of inertia

- **Types of I:-**

1) Area Moment of inertia: Resistance of a rigid body to a change in it's cross-sectional area.

2) Polar Moment of inertia:- Resistance of the Rigid body to a change in it's twist angle.

3) Mass Moment of inertia:- Resistance of a Rigid body to a Change in it's Angular velocity.

$$\sum \vec{M}_o = I_o \alpha$$

$$[N.m] = I_o \, [rad/s^2]$$

$$I_o \Rightarrow [N.m.s^2] \Rightarrow [Kg.\frac{m}{s^2}.m\,s^2] = [Kg.m^2]$$

$$I_o \Rightarrow [Kg.m^2] \quad \text{or} \quad I_o = slug.ft^2$$

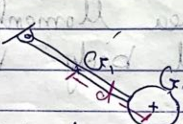
Parallel axis Theorem :-

$$I_o = I_G + md^2$$

* دا ح أقل ما يمكن
لأنه أسهل احتى تلف الجسم
حول مركزه.

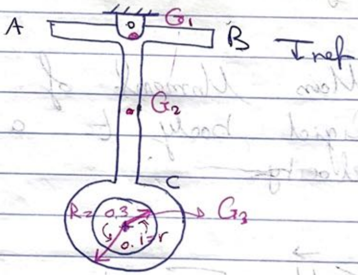
* for Composite Bodies :-
You can add / substract I_o if and only if They were around The same point.

So either $I_{G_1} + (I_{G_2})$ around G_1



or opposite or around a Third neutral axis.

17-19 :-



OC = 1.5 m
AO = OB = 0.4 m

$Mass_r = 3$ Kg/m

$Mass_p = 12$ Kg/m²

① find y_G ② I_G ③ $(I_G)_{rod} = \frac{1}{12} m L^2$

$(I_G)_{plate} = \frac{1}{2} m r^2$

$$y_G = \frac{\sum m_i y_{G_i}}{\sum m_i} = \frac{m_1 y_{G_1} + m_2 y_{G_2} + m_3 y_{G_3}}{m_1 + m_2 + m_3}$$

$m_1 = (0.8)(3) = 2.4$ Kg

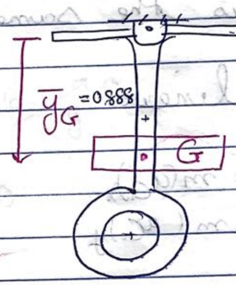
$m_2 = 1.5 \times 3 = 4.5$ Kg

$m_3 = 12 \left[\pi (R^2 - r^2)\right]$

$= 12 \times 3.14 \left((0.3)^2 - (0.1)^2\right) = 3.08$ Kg

$$\bar{y}_G = \frac{0 + (4.5)(0.75) + (1.8)(3.08)}{(2.4)(4.5)(3.08)}$$

$\bar{y}_G = 0.888$ m



2) $I_{G_1} = \frac{1}{12} m_1 L_1^2 = \left(\frac{1}{12}\right)(2.4)(0.8)^2 =$

$I_{G_2} = \frac{1}{12} m_2 L_2^2 = \left(\frac{1}{12}\right)(4.5)(1.5)^2$

$I_{G_3} = \left(\frac{1}{2}\right) m_{P_1} R^2 - \frac{1}{2} m_{P_2} r^2$

$= \frac{1}{2}(12)(\pi)(0.3)^4 - \left(\frac{1}{2}\right)(12)(\pi)(0.1)^4$

$I_G = I_{G_1} + m_1 (0.888)^2 + I_{G_2} + m_2 (0.888 - 0.75)^2$

$+ I_{G_3} + m_3 (0.3 + 1.5 - 0.888)^2$

$\boxed{I_G = 5.61 \text{ kg.m}^2}$

* Radius of Gyration :-

$K_G = \sqrt{\frac{I_G}{m}}$

$\boxed{I_G = m\ K_G^2}$

Rigid 16 17 — Particle 12-15

Force Analysis — Force & Acc 17 — Tran Rot General

No Forces Analysis 16

→ 17.3 : EOM : Translation

17.3

$\alpha = 0$

a is the same at all points

Rectilinear

$\Sigma F_x = m(a_G)_x$

$\Sigma F_y = m(a_G)_y$

$\Sigma M_G = 0$

$\Sigma M_A = m a d$

Curvilinear

$\Sigma F_t = m(a_G)_t$

$\Sigma F_n = m(a_G)_n$

$\Sigma M_G = 0$ at each

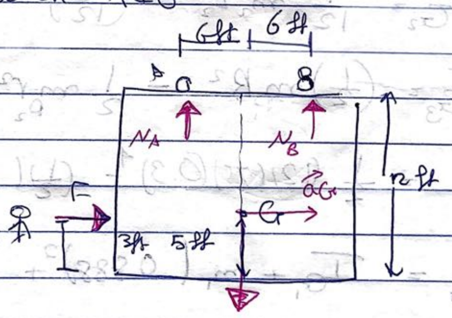
$\Sigma M_A = m a d$

d : distance between A & G

12-24 :

W = 200 Ib

Find F → 12 ft to the right in 5 s starting from rest



Find the Roller reactions at A, B

$$\Delta S = V_1 t + \frac{1}{2} a t^2$$

$$\cancel{8}\,12 - 0 = 0 + \frac{1}{2} a (5)^2$$

$$\boxed{a = 0.96 \ ft/s^2}$$

$$\Sigma F_x = F = m(a_G)_x$$

$$F = \left(\frac{200}{32.2}\right)(0.96)$$

$$\boxed{F = 5.96 \ lb}$$

$$\Sigma F_y = N_A + N_B - W = m\cancel{(a_G)_y}^{\,0}$$

$$N_A + N_B = 200 \quad \text{—— ①}$$

$$\Sigma M_G = 0$$

$$(F)(2) + (N_B)(6) + {}^-(N_A)(6) = 0$$

$$(2)(5.96) + 6\,(N_B - N_A) = 0$$

$$N_B - N_A = -\frac{2 \times 5.96}{6} \quad \text{—— ②}$$

solving ① and ②

$N_B = 99 \ lb \ / \ N_A = 101 \ lb$

17- 43

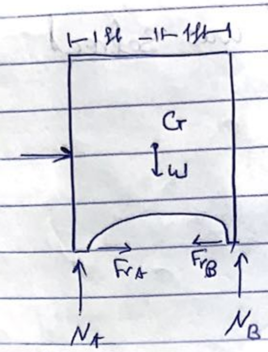
Find a_G, N_A, N_B

$W = 150$ Ib

$P = 30$ Ib

$\mu_s = 0.2$

$\mu_k = 0.15$



Find P_{nec} to overcome the static friction

$$P_{nec} - \mu_s N_A - \mu_s N_B = 0$$

$$N_A + N_B - W = 0$$

$$\Sigma M_G = 0$$

$$-(P)(0.5) - \mu_s N_A (3.5) - \mu_s N_B (3.5) - N_A + N_B = 0.$$

$\underline{P_{nec} = 30 \text{ Ib}}$ $\quad N_A = 15$ lb $\quad N_B = 135$ Ib

$$\left.\begin{array}{l} P_{nec} < P \\ N_{A,B} > 0 \end{array}\right\} \Rightarrow \text{translation.}$$

$$\Sigma F_x = P - \mu_k N_A - \mu_k N_B = m a_{Gx}$$

$$= 35 - 0.15 N_A - 0.15 N_B = \frac{150}{32.2} a_{Gx}$$

$$\Sigma F_y = N_A + N_B - W = 0$$

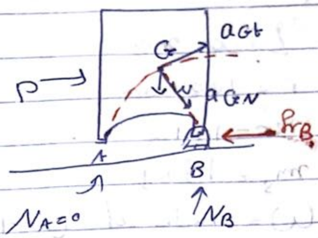
$$\Sigma M_G = -P(0.5) - \mu_k N_A (3.5) - \mu_k N_B (3.5) - N_A + N_B = 0$$

$N_A = 26.9$ Ib

$N_B = 123.1$ Ib

$a_{Gx} = 2.68$ ft/s^2

$P_{nec} > P \Rightarrow$ Rotation

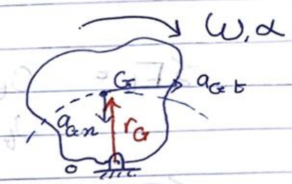


17-4 :- Equation of Motion

Rotation about a fixed axis

$\sum F_t = m\, a_{Gt}$

$= m\, \alpha\, r_G$

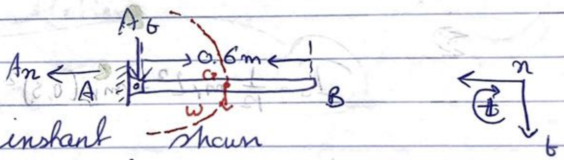


$\sum F_n = m a_{Gn} = m \omega^2 r_G$

$\sum M_G = I_G \alpha$

$\sum M_o = I_o \alpha$

17-65



$m = 12\ kg$

$\omega = 5\ rad/s$ at the instant shown

Find the reactions at A & α

$\sum F_t = A_t + W = m(\alpha\, r_G)$

$\sum F_n = A_n = m\omega^2 r_G$

$A_n = 90\ N.$

$\sum M_A = I_A \alpha$

$W(0.3) = I_A\, \alpha$

$I_A = \frac{1}{12} m L^2 + (m)(0.3)^2$

$\alpha = 24.5\ rad/s^2$

$A_t = -29.4\ N = 29.4 \uparrow N$

• 17-71:

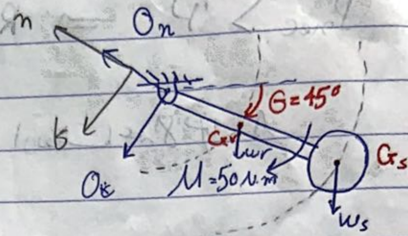
$L = 600\ mm$

$r = 100\ mm$

$m_r = 10\ kg$

$m_s = 15\ kg$

$\omega = 3\ rad/s$ at $\theta = 45°$



$I_{G\,rod} = \frac{1}{12} m L^2$

$I_{G\,sphere} = \frac{2}{5} m r^2$

Find the reactive forces at O:

$\sum F_t = O_t + W_r \cos 45 + W_s \sin 45 = m_r \alpha r_{Gr} + m_s \alpha r_{Gs}$

$O_t \to ?$

$\sum F_n = O_n - W_r \sin 45 - W_s \sin 45 = m_r \omega^2 r_{Gr} + m_s \omega^2 r_{Gs}$

$O_n = 294.92\ N$

$\sum M_O = 50 + (W_r \cos 45)(0.3) + (W_s \cos 45)(0.6 + 0.1) = I_O \alpha$

$I_O = \frac{1}{12} m_r L^2 + m_r (0.3)^2 + \frac{2}{5} m_s r^2 + m_s (0.7)^2$

and $\alpha = 16.68\ rad/s^2$ $\quad I_O =$

$O_t = 51.81\ N$

17.5 :- Equation of Motion: General planar Motion

lec ①

Remember that

For Translation :

$\Sigma F_x = m a_{Gx}$

$\Sigma F_y = m a_{Gy}$

$\Sigma M_G = 0$

$\Sigma M_A = m a d$

For Rot about a fixed axis :-

$\Sigma F_t = m \alpha r_G$

$\Sigma F_n = m \omega^2 r_G$

$\Sigma M_G = I_G \alpha$

$\Sigma M_o = I_o \alpha$

Now :-

For General Planar Motion

$\Sigma F_x = m a_{Gx}$

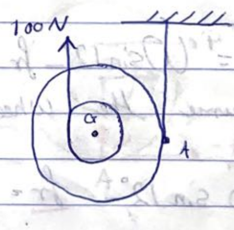
$\Sigma F_y = m a_{Gy}$

$\Sigma M_G = I_G \alpha$

$\Sigma M_{IC} = I_{IC} \alpha$

$\Sigma M_A = m a_G d + I \alpha$

Example :-

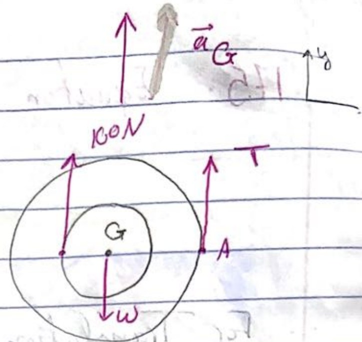


$r = 0.2\,m$

$R = 0.5\,m$

$m = 8\,Kg$

$K_G = 0.35$ Find α



$\Sigma F_x = 0$

$\Sigma F_y = T + 100 - W = m a_{Gy}$

$\rightarrow T + 100 - (8 \times 9.81) = 8(a_{Gy})$

$\Sigma M_A = I_A \alpha$

$=$

$W(0.5) - 100(0.7) = I_A \alpha$

$I_A = I_G + md^2$

$= m k_G^2 + m(0.5)^2$

So $\alpha = 10.3 \text{ rad/s}^2$

To find T

$a_{Gy} = |\alpha| \times |r_{G/A}| = (10.3)(0.5) = 5.15 \text{ m/s}^2$

17-94:-

$r = 1.25$ ft

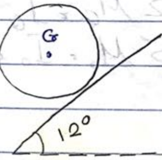
$W = 30$ lb

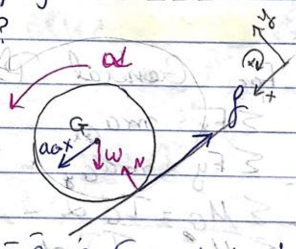
$K_G = 0.6$ ft

$\mu_s = 0.2$ is it Rolling with slipping or without slipping?

$\mu_k = 0.15$

Find α





$\Sigma M_G = I_G \alpha$

same direction

لأنه لازم يكون فيه قوة
تسولي M نفس اتجاه α
علأنه W N ما يسوو M

جميل

$\Sigma F_x = W \sin 12° - f_r = m a_{Gx}$

Assume the wheel rolls without slipping :-

$30 \sin 12° - f_r = \frac{30}{32.2} a_{Gx}$ — ①

$\Sigma F_y = N - W \cos 12° = 0$

$N = 30 \cos 12 = 29.34$ Ib

$\sum M_G = +(f_r)(1.25) = I_G(\alpha)$

$$(f_r)(1.25) = \left(\frac{30}{32.2}\right)(0.6)^2\ (\alpha)$$

$\boxed{a_{Gx} = \alpha r}$ assumption

$f_r = 1.17\ Ib$
$\alpha = 4.35\ rad/s^2$
$a_{Gx} = 5.44\ ft/s^2$

We cal $\mu_s N$ and we make sure it's more than calculated f_r $(\,|f_r| \leq \mu_s N$

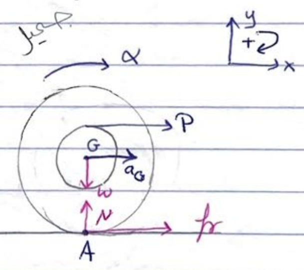
$\mu_s N = (0.2)(29.34) = 5.868$

so: $f_r < \mu_s N$ There is No slipping

But if there is slipping (Assumption is wrong)
→ Then I put $\underline{\underline{\mu_k N}}$ insteal of f_r

(we change each equation)

→ and $a_{Gx} = \alpha r$ Becomes wrong sine the point down is not an $\underline{\underline{IC}}$ just



17-96

$r = 250\ mm$ $\mu_k = 0.15$
$R = 400\ mm$ $P = 50\ N$
$m = 100\ kg$ find α
$k_G = 0.3m$
$\mu_s = 0.2$

$M_G = I_G \alpha$

Assume No slipping

$\Sigma F_x = P + fr = m a_{G_x}$

$\Sigma F_y = N - W = 0$

$N = 981 \, N$

$a_{Gx} = I_G \alpha$

$P(0.25) - fr(0.4) = m k_G^2 \alpha$

$\alpha = 1.3$ rad/s

$a_{Gx} = 0.52 \, m/s^2$

$f_r = 2N$

$N = 981$

$\mu_s N = (0.2)(981)$

$|fr| \overset{??}{\leq} \mu_s N$

$2 \overset{??}{\leq} (0.2)(981)$ ✓ Assumption is right

17-105

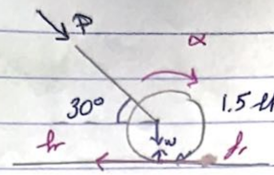
$W = 50\,Ib$

$\mu_s = 0.25$

$I_G = \frac{1}{2} m r^2$

P_{max} for No slipping

α



$\Sigma F_x = P\cos 30 - fr = m\,a_{Gx}$

$\Sigma F_y = N - W - P\sin 30 = m\,a_{Gy}$

$\Sigma M_G = \mu_s N(1.5) = I_G\,\alpha$

$a_{Gr} = \alpha r$

$\alpha = 18.93\ rad/s^2$

$P = 76.4\ Ib$

$N = 88.81\ Ib$

$a_{Gx} = 28.39\ ft/s^2$

Chapter 18: Kinetics of a Rigid Body : Work & Energy

$$T_1 + \sum U_{1-2} = T_2$$
$$T_1 + V_1 = T_2 + V_2$$

→ $T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$ → Rotation about a fixed axis + General planar motion

→ $V_g = mgh_G$

→ $V_e = \frac{1}{2} k s^2$

$$U_{F_{1-2}} = F \cos\theta \, s$$

$$U_{W_{1-2}} = -W(y_2 - y_1) = -W \Delta y$$

$$U_{s_{1-2}} = -\frac{1}{2} k (s_2^2 - s_1^2)$$

$$U_{fr_{1-2}} = -f_r \, s$$

* Friction force: → No slipping → No Work
  → slipping → Work is calculated

Rot & Translation

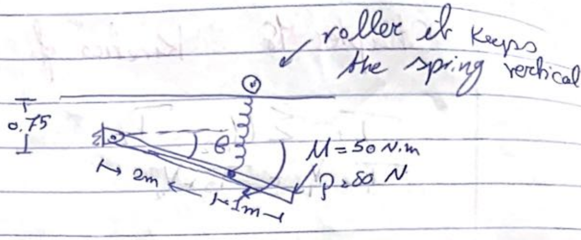
* Work of a Couple Moment

$$U_{M_{1-2}} = \int_{\theta_1}^{\theta_2} M \, d\theta$$

$$U_{M_{c\,1-2}} = M(\theta_2 - \theta_1)$$
rad

Example

Find the Total Work Done

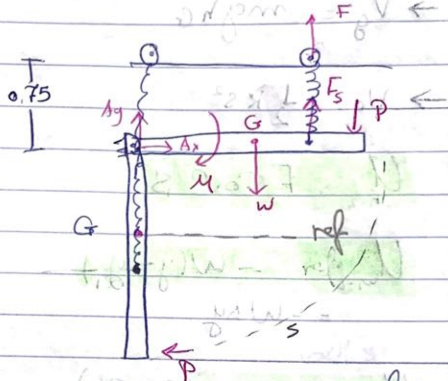


$m = 10\ kg$

$\theta_1 = 0$ to $\theta_2 = 90$

$k = 30\ N/m$

$l_{un} = 0.5\ m$



$$U_{M_{1-2}} = \int_{\theta_1}^{\theta_2} M\, d\theta$$

$$= \int_0^{90} 50\, d\theta$$

$$= 50 \frac{\pi}{2} = 78.5\ J$$

$$U_{W_{1-2}} = -W(y_2 - y_1)$$

$$= -(10)(9.81)(0 - 1.5)$$

$$= 147.5\ J$$

$s_1 = l_{st_1} - l_{unst}$

$s_1 = 0.75 - 0.5 = 0.25$

$s_2 = l_{st_2} - l_{unst}$

$= 2.75 - 0.5$

$s_2 = 2.25$

$$U_{S_{1-2}} = -\frac{1}{2} k (s_2^2 - s_1^2)$$

$$= \left(-\frac{1}{2}\right)(30)(2.25^2 - 0.25^2)$$

$$= -75\ J$$

$$U_{P_{1-2}} = (80)(3)\left(\frac{\pi}{2}\right)(1)$$

$$= 377\ J$$

$$U_T = \Sigma U = 528\ J$$

2

18-9

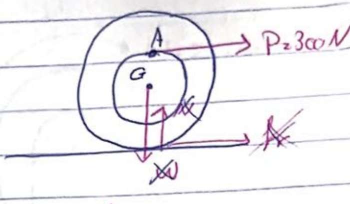
$r = 0.2m$

$R = 0.4m$

$k_G = 0.275m$

$m = 100\ Kg$ starting from rest

rolling without slipping   friction does no wor



• Find ω_2 when G has moved 1.5 m to the right.

$T_1 + \sum U_{1-2} = T_2$

$V_G = \omega r_{G/IC} = \frac{S_G}{t}$

$V_A = \omega r_{A/IC} = \frac{S_A}{t}$

$T_1 = 0$

$T_2 = (\frac{1}{2}) m V_G^2 + \frac{1}{2} I_G \omega^2$???

$\cancel{U_{P_{1-2}} = (300)(\frac{1.5}{12}) = 450\ J}$

$\frac{r_{A/IC}}{r_{G/IC}}$

$\cancel{450 = 50 V_G^2 + (\frac{1}{2})}$

$\frac{S_G}{S_A} = \frac{r_{G/IC}}{r_{A/IC}}$

$U_{P_{1-2}} = (300) S_A$

$= \frac{(300)(1.5)(0.4 + 0.2)}{0.4}$

$= 675\ J$

$675 = (\frac{1}{2})(100) V_G^2 + (\frac{1}{2})((100)(0.275)^2)\omega_2^2$

$675 = 50\ \omega^2 (0.4)^2 + (50)(0.275)^2 \omega_2^2$

$\omega_2 = 7.57\ rad/s$

lec 2 ...

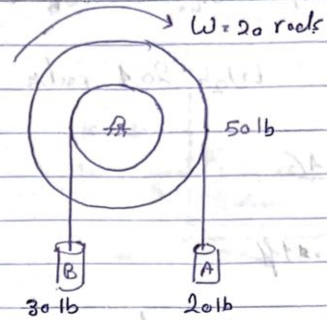
18-8.

$r = 0.5\ ft$

$R = 1\ ft$

$K = 0.6\ ft$

- Find ω_2 after A has moved 2 ft down



$$T_1 = \frac{1}{2} m_A V_{A_1}^2 + \frac{1}{2} m_B V_{B_1}^2 + \frac{1}{2} m V_{o_1}^2 + \frac{1}{2} I_o \omega_1^2$$

$$= \left(\frac{1}{2}\right)\left(\frac{20}{32.2}\right)(20 \times 1)^2 + \left(\frac{1}{2}\right)\left(\frac{30}{32.2}\right)(20 \times 0.5)^2 + \cancel{\left(\frac{1}{2}\right)\left(\frac{50}{32.2}\right)(0)} + \left(\frac{1}{2}\right)(0.6)^2\left(\frac{50}{32.2}\right)(20)^2$$

$$T_1 = 282.61\ ft.lb$$

$$T_2 = \frac{1}{2} m_A V_{A_2}^2 + \frac{1}{2} m_B V_{B_2}^2 + \cancel{\frac{1}{2} m V_{o_2}^2} + \frac{1}{2} I_o \omega_2^2$$

$$= \frac{1}{2} m_A (\omega_2 \times 1)^2 + \frac{1}{2} m_B (\omega_2 \times 0.5)^2 + \frac{1}{2} I_o \omega_2^2$$

$$T_2 = 0.7065\ \omega^2$$

$S_A = 2\ ft$

$R\theta_A = 2$

$\theta_A = 2\ rad$

$S_B = r\,\theta_B$

$= (0.5)(2) = 1\ ft$

$$U_{W_A} = -W_A\,(y_{A_2} - y_{A_1})$$

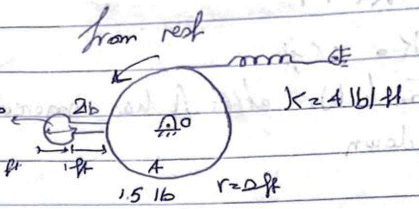
$$= -20\,(-2) = 40\ ft.lb$$

$$U_{W_B} = -30\,(1) = -30\ ft.lb$$

$T_1 + U_{W_A} + U_{W_B} = T_2$

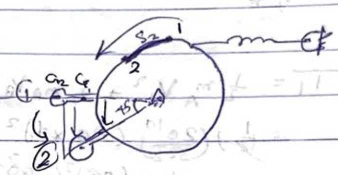
$\omega_2 = 20.4$ rad/s

18-46



$s_1 = 1$ ft

Find ω_2 at $\theta = 45°$



$T_1 = 0$

$T_2 = \frac{1}{2} m_s v_{s_G}^2 + \frac{1}{2} I_s \omega^2$

$+ \frac{1}{2} m_r v_{r_G}^2 + \frac{1}{2} I_r \omega_G^2$

$+ \frac{1}{2} m_d v_{d_G}^2 + \frac{1}{2} I_d \omega_{d_G}^2$ $\omega^2 \times 2.5^2$

$= (\frac{1}{2}) m_s (3.5\omega_2)^2 + \frac{1}{2}$ ———

$T_2 = 2.5802\, \omega_2$

$V_1 = V_{e_1} + V_{g_1}$

$V_1 = \frac{1}{2} k s_1^2 + 0 = \frac{1}{2} \times 4 \times 1^2 = 2$ lb.ft

$V_2 = V_{e_2} + V_{g_2}$ $s_2 = 1 + \frac{\pi}{4} \times 2$

$= (\frac{1}{2})(4)(s_2)^2 + \cdots$ $=$

$V_{e_2} = 13.218$ lb.ft

$V_{g_2} = -m_r g\, 2.5 \sin 45 - m_s g\, 3.5 \sin 45 = -28.284$ lb.ft

$T_1 + V_1 = T_2 + V_2$

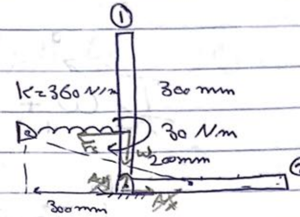
$\omega_2 =$

Final Question:

$m = 8\ Kg$

at ① Spring is unstretched

starting from rest final ω_2



$I_G = \frac{1}{12} m L^2$

$T_1 = 0$

$T_2 = \frac{1}{2} m V_{G_2}^2 + \frac{1}{2} I_G \omega_2^2$

$= (\frac{1}{2})(8)(\omega_2 \times 2.5)^2 + (\frac{1}{2})(\frac{1}{12} m L^2)\omega_2^2$

$U_{W_{1-2}} = -W(y_2 - y_1)$

$= -8 \times 9.81 (0 - 0.25)$ J

$U_{M_{1-2}} = 30(\frac{\pi}{2}) = 15\pi$ J

$U_{s_{1-2}} = -\frac{1}{2} k (S_2^2 - S_1^2) = -\frac{1}{2} k (\sqrt{500^2 + 200^2} - 300)^2$