Chapter 19: Kinetics of a Rigid Body: Impulse & Momentum

$$m\vec{V}_{G_1} + \int_{t_1}^{t_2} \Sigma \vec{F}\, dt = m\vec{V}_{G_2}$$ Principle of momentum and impulse
Translation

- Conservation of linear Momentum (Impact)

$$\left(\Sigma m_i V_{G_i}\right)_1 = \left(\Sigma m_i V_{G_i}\right)_2$$

$$\vec{H}_{o_1} + \int_{t_1}^{t_2} \Sigma \vec{M}_o\, dt = \vec{H}_{o_2}$$

$\vec{H}_{o_1} = m V_{G_1} d + I_G \omega_1$ d: perpendicular distance Between O and G

- Conservation of Angular Momentum:-

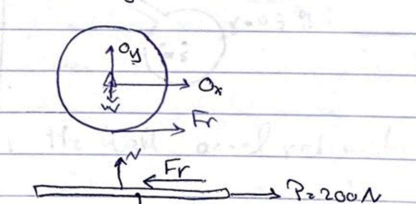
$\Sigma \vec{M}_o = 0$

$\vec{H}_{o_1} = \vec{H}_{o_2}$

19-10:-

$K_o = 125$ mm

Find t to get $\omega = 20$ rad/s starting from rest



For the rack (translation)

x-axis

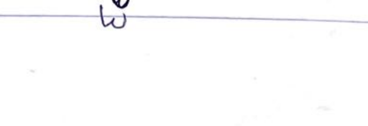
$$\cancel{mV_1}^{0} + \int_0^t (P - Fr)\, dt = mV_2$$

$(P - Fr)t = m V_2$

$(200 - Fr)t = 20 V_2$ — ①

$(200 - Fr)t = (20)(20 \times 0.15)$

$(200 - Fr)t = 60$ — ①



for the

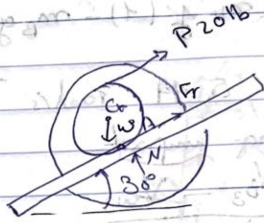
Angular Moment:

$$\cancel{\vec{H}_{G_1}}^{0} + \int_0^t \sum \vec{M}_G \, dt = \vec{H}_{G_2} \qquad G \equiv 0$$

$$F_r(0.15)\,t = m\,\cancel{V_{G_2}\,d} + I_G\,\omega_2$$

$$t\,F_r(0.15) = (30)(0.125)^2(20) \quad —②$$

$t = 0.6\,s$



19-20

rolls up without slipping

Find ω_2 after 5 s

Starting from rest

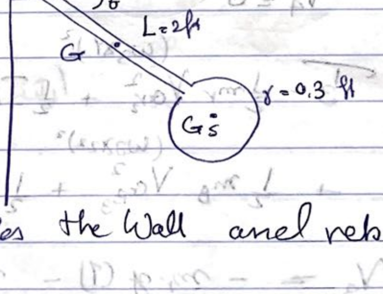
$$\vec{H}_{A_1} + \int_0^t \sum \vec{M}_A \, dt = \vec{H}_{A_2}$$

$$\int_0^5 (20 \times 1.5 - 100\sin 30 (0.5))\,dt = m V_{G_2} d_{G-A} + I_G \omega_2$$

$$25 = \frac{100}{32.2}\,(V_{G_2})(0.5) + \frac{100}{32.2}(0.75)^2\,\omega_2$$

$(\omega_2)(0.5)$

$\omega_2 = 9.91$ rad/s



19-47

$W_B = 10$ lb

$W_r = 4$ lb

released from rest at $\theta = 0$

Find θ_{max} after the ball strikes the Wall and rebounds

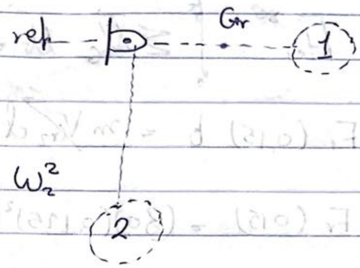
$e = 0.6$

Before impact :-

$T_1 + V_1 = T_2 + V_2$

$T_1 = 0$

$V_1 = 0$



$$T_2 = \left(\tfrac{1}{2}\right)(m_r)\, V_{Gr_2}^2 + \tfrac{1}{2} I_{Gr}\, \omega_2^2 + \tfrac{1}{2} m_B\, \underset{(\omega_2 \times 2.3)}{V_{GB_2}^2} + \tfrac{1}{2} I_{GB}\, \omega_2^2$$

$(1\omega_2)^2$

$V_2 = - m_r g\,(1) - m_B g\,(2.3)$

$\omega_2 = 5.44$ rad/s } Just before impact

→ During impact

$$e = \frac{V_{B_3} - \cancel{V_{w_3}}}{\cancel{V_{w_2}} - V_{B_2}}$$

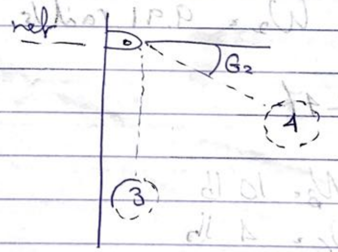
$$0.6 = \frac{V_{B_3}}{(5.44)\cancel{\times}(2.3)} \Longrightarrow V_{B_3} = \omega_3 \times 2.3$$

So $\omega_3 = -3.27$ rad/s rot counter clockwise

After impact

$T_3 + V_3 = T_4 + V_4$

$T_4 = 0$



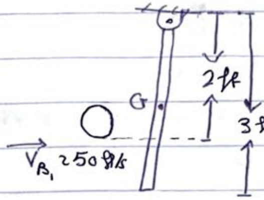
$$T_3 = \tfrac{1}{2} m_r\, \overset{(\omega_3 \times 1)^2}{V_{Gr_2}^2} + \tfrac{1}{2} I_{Gr}\, \omega_3^2 + \tfrac{1}{2} m_B\, \overset{(\omega_3 \times 2.3)^2}{V_{GB_3}^2} + \tfrac{1}{2} I_{GB}\, \omega_3^2$$

$V_3 = - m_r g\,(1) - m_B g\,(2.3)$

$V_4 = -m_r g(1 \sin\theta_2) - m_s g(2.3 \sin\theta_2)$

so $\theta_2 = 39.8^\circ$

19-50:-



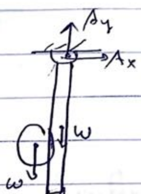
$W_r = 6$ lb

rod is originally at rest

$W_B = 1$ lb

$e = 0.7$

Find ω_2 after impact



$\sum H_{A_1} = \sum H_{A_2}$

$H_{AB_1} + \cancel{H_{Ar_1}}^{0} = H_{AB_2} + H_{Ar_2}$

$m_B V_{B_1} d_{B-A} = m_B V_{B_2} d_{B-A} + m_r V_{Gr_2} d_{G-A} + I_{Gr}\omega_2$

$\left(\frac{1}{32.2}\right) \overset{50}{V_{B_1}} (2) = \left(\frac{1}{32.2}\right) V_{B_2} (2) + \left(\frac{6}{32.2}\right)(\underbrace{\bullet}_{V_{Gr_2}})(1.5) + \left(\frac{1}{12}\right)\left(\frac{6}{32.2}\right)(3)^2 \omega_2$

$V_{Gr_2} \to \omega_2(1.5)$

$0.7 = \frac{V_{B_2} - V_{Gr_2}}{V_{Gr_1} - V_{B_1}}$

$0.7 = \frac{(V_{B_2}) - (\omega_2)(1.5)}{0 - 50}$

$\omega_2 = 7.73$ rad/s

$V_{B_2} = -19.5$ ft/s → ارتدت الطابة