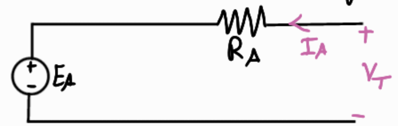
DC Machines

Generators

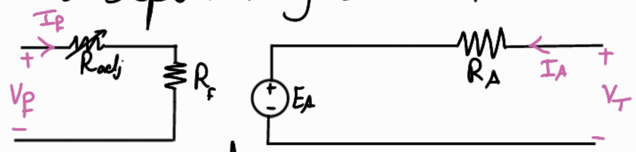
- Permanent Magnet
- Separately Excited
- Shunt
- Series
- Compound

Motors

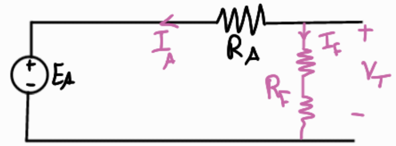
- Permanent Magnet



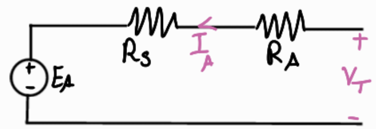
- Separately Excited



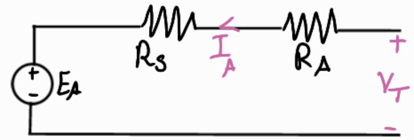
- Shunt



- Series



- Compound



Chapter 1 Formulas

$f_m = \frac{\omega_m}{2\pi}$ $n_m = 60 f_m$

$H = \frac{Ni}{l_c}$ H: intensity [A.N/m]

$B = \mu H$ B: density [T]

$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ for Air

$\mu_r = 2000 - 6000$ for Metal

$\phi = \frac{\mu N i}{l_c} A = BA = \frac{\mathcal{F}}{\mathcal{R}}$ [weber]

$\mathcal{F} = Ni$ [mmf]

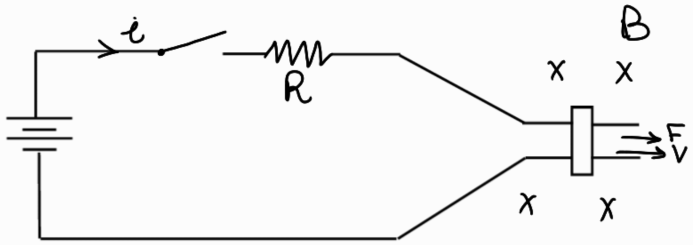
$\mathcal{R} = \frac{l_c}{\mu A}$ [A.N/weber]

Faraday's law: $e_{ind} = -N \frac{d\phi}{dt}$

linear DC Machine

$i = \frac{V_B - e_{ind}}{R}$

$F = i\,(l \times B)$



If a load is introduced

$e_{ind} = V_B - iR$ (opposite Direction)

$e_{ind} = V_B + iR$ (Same Direction)

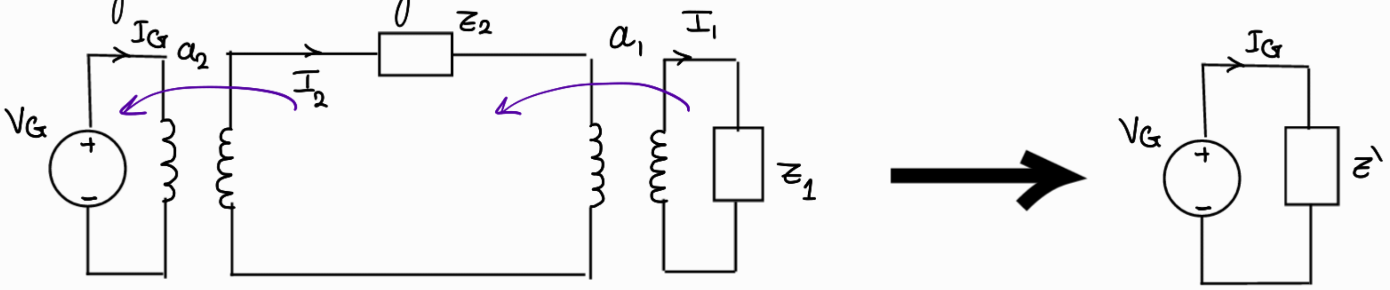
where $i = \frac{F}{lB}$

$V_{\text{steady state}} = \frac{e_{ind}}{lB}$

At no load $e_{ind} = V_B$

Chapter 2 Formulas (Transformers)

Reflection in general



Equations:

$$Z_1' = a_1^2 Z_1$$

$Z_{eq} = Z_2 + Z_1'$ ⟶ Reflecting this

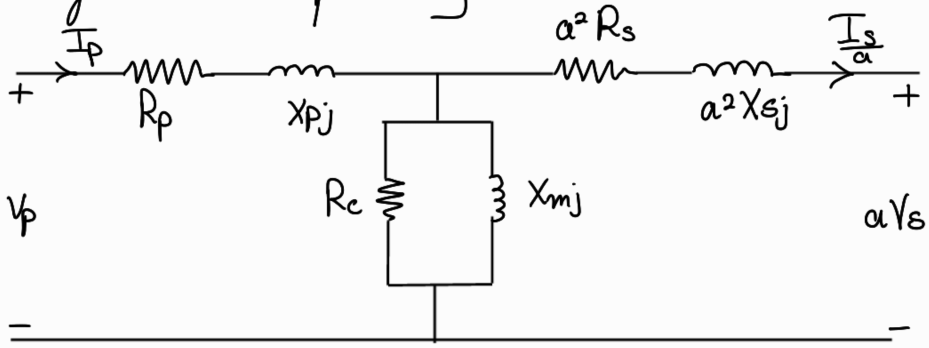
$$Z' = a_2^2 Z_{eq}$$

$I_G = \frac{V_G}{Z'}$ ⟶ Reflecting this Back

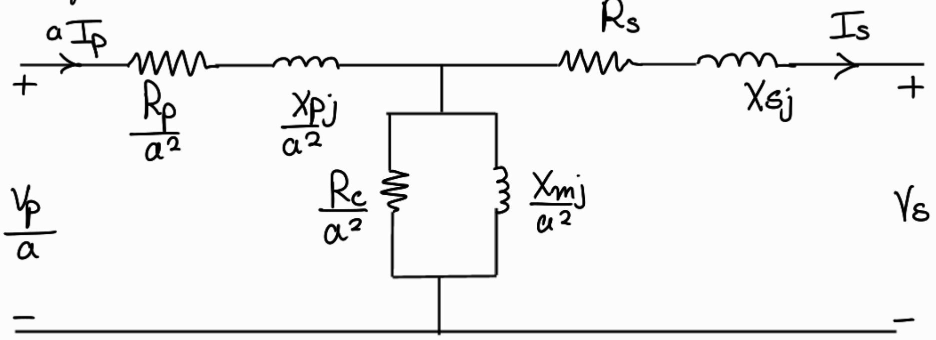
$I_2 = \frac{1}{a_2} I_G$, $I_1 = \frac{1}{a_1} I_2$

One transformer reflection

Referred to primary

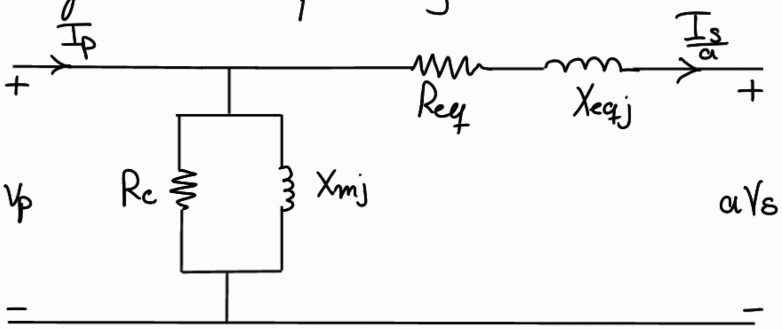


Referred to secondary



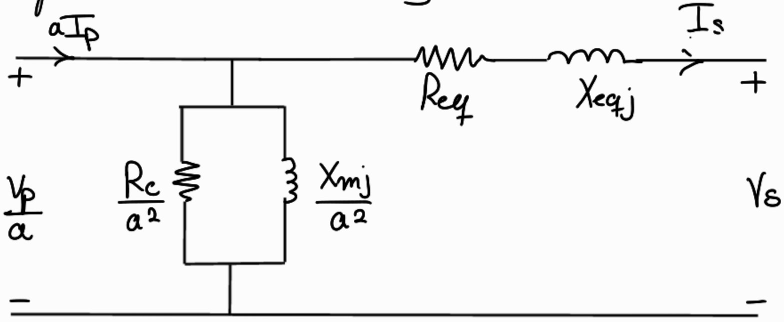
Short of these circuits

Referred to primary



$$R_{eq} = R_p + a^2 R_s$$
$$X_{eq}j = X_p + a^2 X_s$$

Referred to secondary



$$R_{eq} = \frac{R_p}{a^2} + R_s$$
$$X_{eq}j = \frac{X_p}{a^2} + X_s$$

Open Circuit test

$$Y = \frac{1}{R_c} + \frac{1}{X_mj} = Y \angle -\theta$$

$$Y = \frac{I_{o.c}}{V_{o.c}}, \quad \theta = \cos^{-1} \frac{P_{o.c}}{V_{o.c} I_{o.c}}$$

↓ phases

Short circuit test

$$Z = R_{eq} + X_{eq}j = \frac{V_{s.c}}{I_{s.c}} \angle \theta$$

$$\theta = \cos^{-1} \frac{P_{s.c}}{I_{s.c} V_{s.c}}$$

Per unit System

$$\text{Quantity per unit} = \frac{\text{Actual value}}{\text{Base value of Quantity}}$$

$$P_{base\,\phi} = Q_{base\,\phi} = S_{base\,\phi}$$

$$I_{base} = \frac{S_{base\,1\phi}}{V_{base\,\phi}}$$

$$Z_{base\,\phi} = R_{base\,\phi} = X_{base\,\phi} = \frac{V_{base\,\phi}}{I_{base\,\phi}} = \frac{(V_{base})^2}{S_{base}}$$

Voltage Regulation

$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s,fl}} \times 100\%$$

$$V_{s,nl} = \frac{V_p}{a} = V_s + I_s (Z_{eq})$$

(I_s if rated $= \frac{S_{rated}}{V_{rated}}$

Transformer efficiency

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

if Given

$P_{out} = S\ PF = V_S I_S \cos\theta$

only R from Z_{eq}

$P_{in} = P_{out} + P_{cu} + P_{core}$

$P_{cu} = (I_S)^2 R_{eq}$

$P_{core} = \frac{(V_P/a)^2}{(R_C/a^2)} = \frac{V_P}{R_C}$

Δ and Y connection

For Δ $\longrightarrow$ $V_{\phi P} = V_L$, $I_{\phi P} = \frac{I_L}{\sqrt{3}}$, $S_{\phi P} = \frac{S}{3}$

For Y $\longrightarrow$ $V_{\phi P} = \frac{V_L}{\sqrt{3}}$, $I_{\phi P} = I_L$, $S_{\phi P} = \frac{S}{3}$

Turns ratio a

- Δ–Δ, Y–Y $\rightarrow$ a
- Δ–Y $\rightarrow$ $\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3} V_{\phi S}} = \frac{1}{\sqrt{3}} a$
- Y–Δ $\rightarrow$ $\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3} V_{\phi P}}{V_{\phi S}} = \sqrt{3}\, a$

Chapter 4 Formulas (Synchronous Generators)

$$n_m = \frac{120 f_e}{P} = n_{sync}$$

Note
$S = \sqrt{3} I_L V_L$
No $\cos\theta$, $\sin\theta$

$$E_A = \sqrt{2}\,\pi N_c \phi f = K \phi \omega$$

E_A → induced voltage

General equation

$$E_A = V_\phi + I_A (R_A + X_s j)$$

Power calculations

θ : Angle Between V_ϕ, I_A
δ : Angle between E_A, V_ϕ

$$P_{in} = \tau_{app}\,\omega_m$$

$$P_{conv} = \tau_{ind}\,\omega_m$$

$$P_{out} = \sqrt{3} V_L I_L \cos\theta \quad \text{or} \quad P_{out} = S \times PF$$

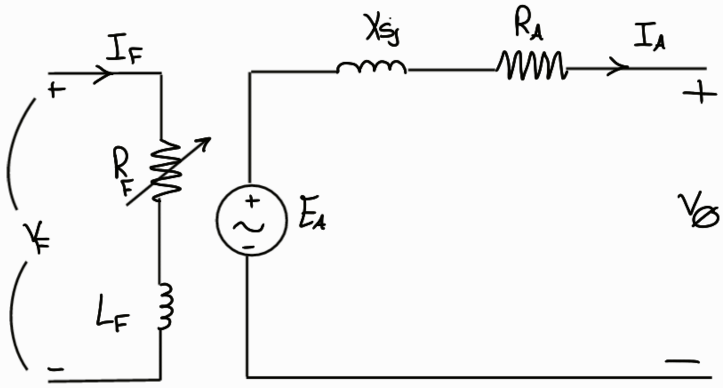
for 3 phase

$$P_{\text{copper losses}} = 3 R_A (I_A)^2$$

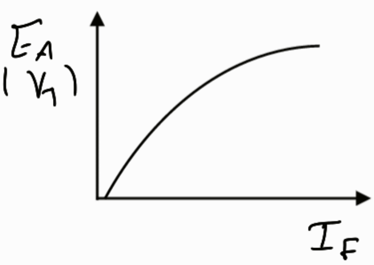
$$\tau_{ind} = \frac{3 V_\phi E_A \sin\delta}{X_s\,\omega_m}$$ Consider $R_A = 0$

Voltage Regulation

$$V_R = \frac{V_{nl} - V_{fl}}{V_{fl}}$$



OCC curve



E_A is Multiplied by $\sqrt{3}$ if Y Before using curve

Chapter 5 Formulas (Synchronous Motors)

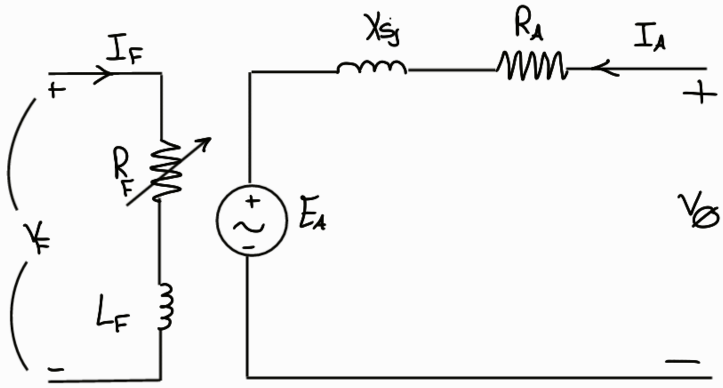
At coupling $n_m = \frac{120 f_e}{P}$

General equation

$V_\phi = E_A + I_A (R_A + X_s j)$

$P = \frac{3 E_A V_\phi \sin \delta}{X_s}$ Assuming $R_A = 0$

$\tau_{pull\ out} = 3 \tau_{rated}$ (full load)



$E_A \sin \delta \propto P = \text{constant}$

$(E_A \sin \delta)_1 = (E_A \sin \delta)_2$

$P_{in} = \sqrt{3} V_L I_L \cos \theta$

Static stability power limit

$P_{max} = \frac{3 V_\phi E_A}{X_s}$

$P_{conv} = P_{IN} - P_{cu}$

$P_{conv} - P_{out} = P_{mech} + P_{core} + P_{stray}$

Note

- If E_A is leading V_ϕ
  $\delta > 0 \rightarrow$ Generator
- If E_A is lagging V_ϕ
  $\delta < 0 \rightarrow$ Motor

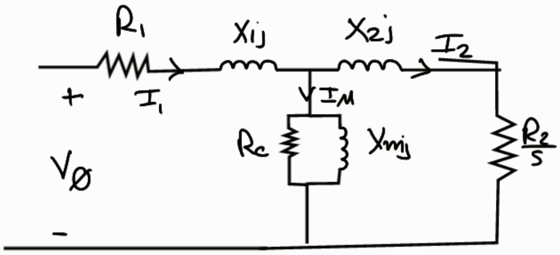
Chapter 6 Formulas (Induction Motors)

$n_{slip} = n_{sync} - n_m$ → $n_m =$ at full load

$n_{sync} = \frac{120 f_e}{P}$

$n_m = (1-S)\, n_{sync}$

$f_r = s f_e = \frac{P}{120} (n_{sync} - n_m)$



$Z_{eq} = R_1 + X_1 j + \left(X_2 j + \frac{R_2}{S} \parallel X_m j \right)$

$I_2 = a_{eff}\, I_1$

Note

If load is increased in ratio X

$S_{new} = X S$

↳ This is used in Speed

Power calculations

$P_{in} = \sqrt{3}\, V_L I_L\, PF$

$P_{SCL} = 3 \underbrace{I_1^2 R_1}_{\text{Stator Side}}$

$P_{RCL} = 3 I_2^2 R_2 = S P_{AG}$

$P_{AG} = P_{in} - P_{SCL} - P_{core} = 3 I_2^2 \frac{R_2}{S}$

$P_{conv} = (1-S) P_{AG}$

$\tau_{ind} = \frac{P_{conv}}{\omega_{sync}}$

$\tau_{load} = \frac{P_{out}}{\omega_m}$

$P_{conv} = P_{in} - P_{SCL} - P_{core} - P_{RCL} - P_{F\&W}$

$P_{out} = P_{conv} - \underbrace{P_{stray}}$

↳ P_{mech}, P_{core}, P_{misc}

$$P_{out} = P_{conv} - P_{mech} - P_{core} - P_{misc}$$

Rotor power factor

$$\theta_R = \tan^{-1}\left(\frac{X_2}{R_2/s}\right)$$

Note

$$3I_A^2 R_F = 3I_2^2 \frac{R_2}{S}$$

↳ parallel of $R_2 + X_2 j$ and $X_m j$

Pullout torque

$$|V_{Th}| = \frac{V_\phi X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}}$$

$$Z_{Th} = Z_{eq} = R_{Th} + X_{Th} j$$

where $Z_{eq} = (R_1 + X_1 j) // (X_m j) = \frac{(R_1 + X_1 j) X_m j}{(R_1 + X_1 j) + X_m j}$

$$s_{max} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}}$$

$$\tau_{max} = \frac{3 V_{Th}^2}{2\omega_{sync}\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}\right]}$$

Note: $\delta = \theta_R + 90°$

$$\sin\delta = \underbrace{\sin(\theta_R + 90°)}_{\cos\theta_R}$$

$\cos\theta_R = PF$ of Rotor

$$\theta_R = \tan^{-1}(sX_{R_0}/R_R)$$

Note:

- changing Frequency will change reactances and V_{rated}

reducing f → increases by factor f_{new}/f_{old} → $X_{new} = \frac{f_{new}}{f_{old}} X_{old}$

increasing f → decreases by factor f_{new}/f_{old} ← same

$$V_{rated} = \frac{f_{new}}{f_{old}} V_{old}$$

Chapter 8 Formulas (DC Machines)

At no load $I_A = 0 \longrightarrow E_A = V_T$

Speed

$$\frac{E_A}{E_{A_0}} = \frac{n}{n_0}$$

241   1200

$$\frac{E_{A_1}}{E_{A2}} = \frac{n_1}{n_2}$$

At full load $\longrightarrow I_A = I_L - I_F$

rated

- If we have magnetization curve & only speed of the curve
  - we find new I_f
  - we use it to obtain E_A from curve $\longrightarrow$ This E_A is at curve speed
  - we use KVL to obtain E_A at I_f found $\longrightarrow$ This E_A is at the speed we want to find

- If we have magnetization curve, but rated speed is different from curve
  - we find new I_f (I_A)
  - we " at " $I_f \longrightarrow E_{A_1} \longrightarrow$ new velocity
  - we use old I_f to find $E_{A_1} \longrightarrow$ old velocity
  - $\frac{E_{A_1}}{E_{A_2}} = \frac{\phi_1}{\phi_2} \frac{n_1}{n_2}$ n_1 = rated But not from curve

    The one we want to find

we use

$I_{f_{old}}$

$$\left.\frac{E_{A_{10}}}{E_{A_{20}}} = \frac{\phi_1}{\phi_2}\right\}$$ at curve speed

I_f new

Speed Regulation

$$SR = \frac{n_{m,nl} - n_{m,fl}}{n_{m,fl}}$$

Series DC motor

$$\omega = \frac{V_T}{\sqrt{\tau_{ind} KC}} - \frac{R_A + R_S}{KC}$$

Shunt DC motor

$$\omega = \frac{V_T}{K\phi} - \frac{R_A \tau_{ind}}{(K\phi)^2}$$

Converted Power

$$P_{conv} = E_A I_A = \tau_{ind} \omega_m$$

$$E_A = I_A X_s j + V_\phi$$

$$\frac{480}{\sqrt{3}}\angle\delta = 60\angle 53.3 + V_\phi \angle 0°$$

$$V_\phi \cos 0 + \cancel{V_\phi \sin 0 j} \to 0$$

$$277\cos\delta + 277\sin\delta j = 60\cos 53.3 + 60\sin 53.3 j + V_\phi$$

$$277\cos\delta = 60\cos 53.3 + V_\phi \longrightarrow eq_1$$

$$\boxed{277\sin\delta = 60\sin 53.3}$$

$$10$$