

## Part ①

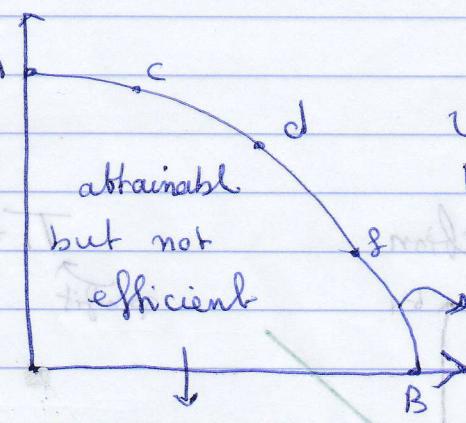
# Engineering Economy

### • Factors of Production:-

- 1 - land
- 2 - labor
- 3 - Capital
- 4 - Entrepreneurs

### • Production possibility

Capital Goods



Frontier :-

unattainable  
but inefficient

attainable and efficient

consumer Goods

- Moving (There is no opportunity Cost) inside

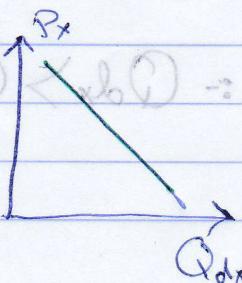
### \* Demand :-

#### ↳ Determinants :-

- 1 - Prices ( $P_x$ )
- 2 - Consumer Taste ( $T$ )
- 3 - income ( $I$ )
- 4 - Number of consumers ( $N$ ) → Substitute goods
- 5 - Price of related Goods ( $P_r$ ) → Complement goods
- 6 - Expectations ( $E$ )

#### ↳ General Demand function:

$$\frac{dQ_{dx}}{dP_x} < 0$$



*Some for Supply*

- If We Change Price Only → Movement on the Curve
- If ~ other factors → Shifting of the Curve

## \* Supply:

↳ Determinants:-

[1] Price

[2] Number of firms

[3] Tech

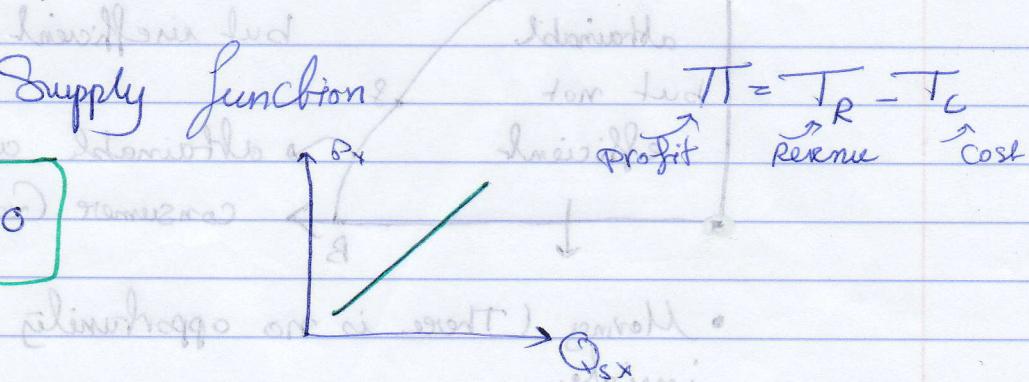
[4] Taxes and Subsidies

[5] Price of other Goods

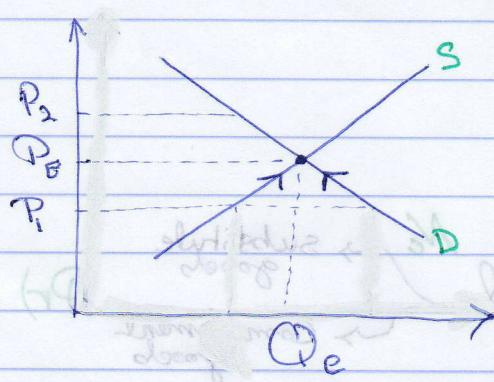
[6] Producers Expectations

↳ General Supply Function

$$\frac{dQ_{sx}}{dP_x} > 0$$



## Equilibrium Point:-



$$Q_{dx} = Q_{sx}$$

If There's no Taxes.

Subtract it from P in Q\_s

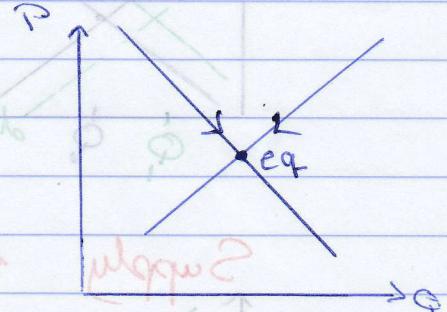
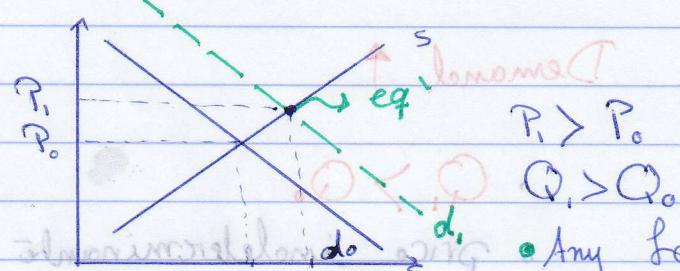
$$\text{at } P_1 \therefore Q_{dx} > Q_{sx}$$

→ demand is more than supply  
so shortage. So Price Increases

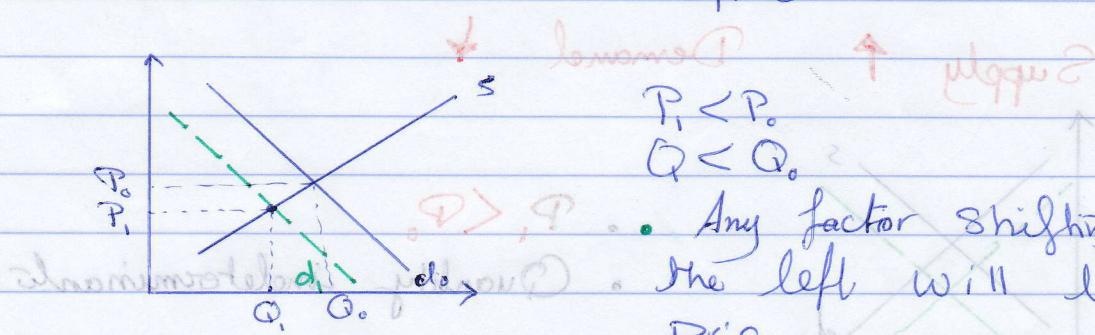
at  $P_2$  :-  $Q_{sx} > Q_{dx}$   $\rightarrow$  demand is less than supply  $\therefore$  Surplus of Goods  
The Price goes Down

### • Cases for shifting Curves:-

$\rightarrow$  Change in demand :-

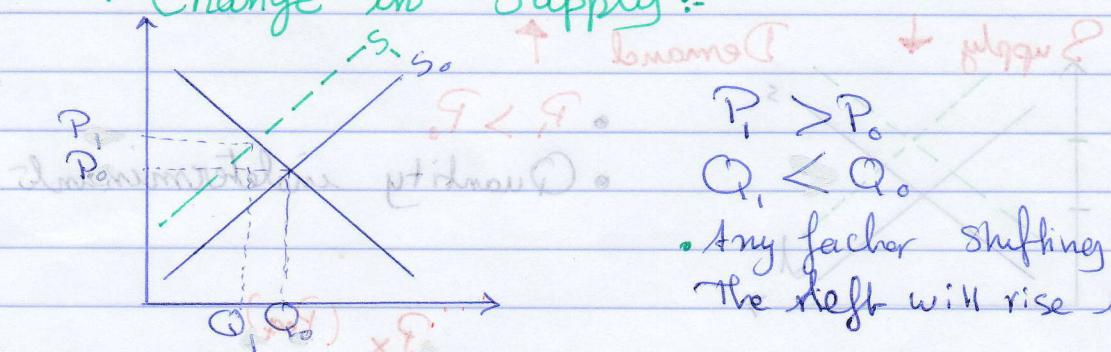


- Any factor shifting D.C to the right will rise the Price.

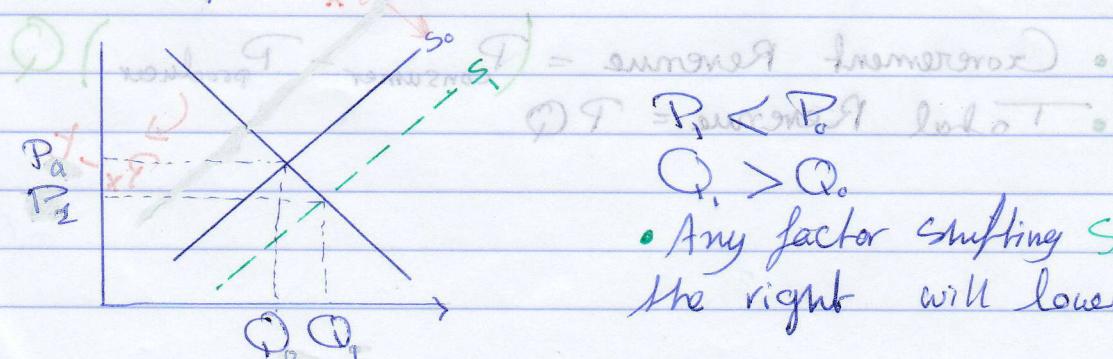


- Any factor shifting D.C to the left will lower the Price.

$\rightarrow$  Change in Supply :-



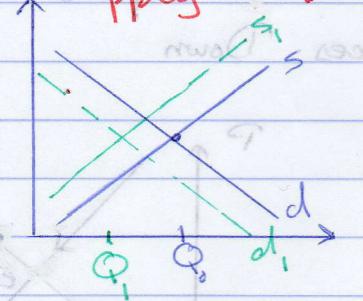
- Any factor shifting S.C to the right will rise the Price



- Any factor shifting S.C to the right will lower the Price

→ Change in Both Sides:-

Supply ↑ Demand ↓

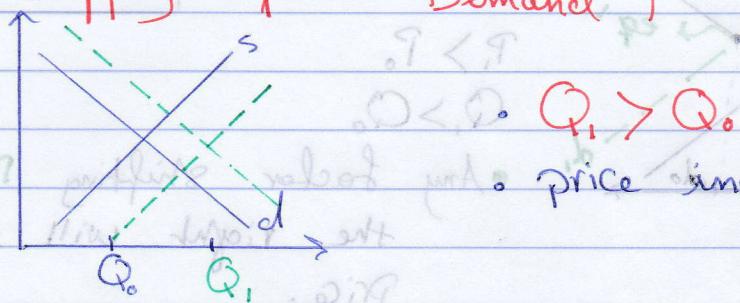


$$Q_1 < Q_0$$

• Price indeterminate

∴ Income in. sign.

Supply ↑ Demand ↑

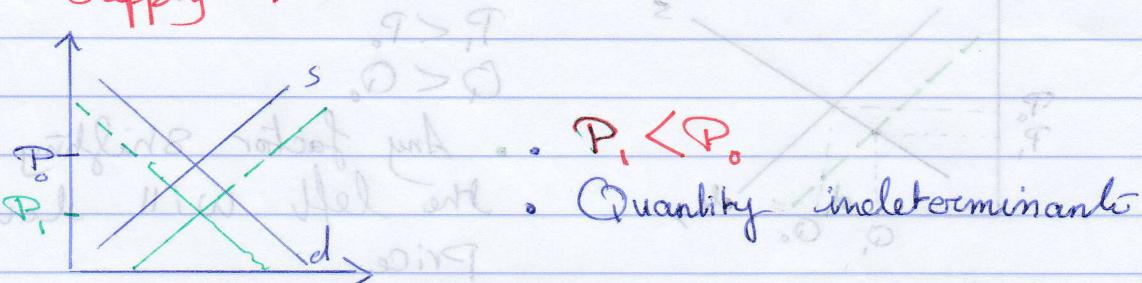


$$P_1 < P_0$$

$$Q_1 > Q_0$$

• Price indeterminate

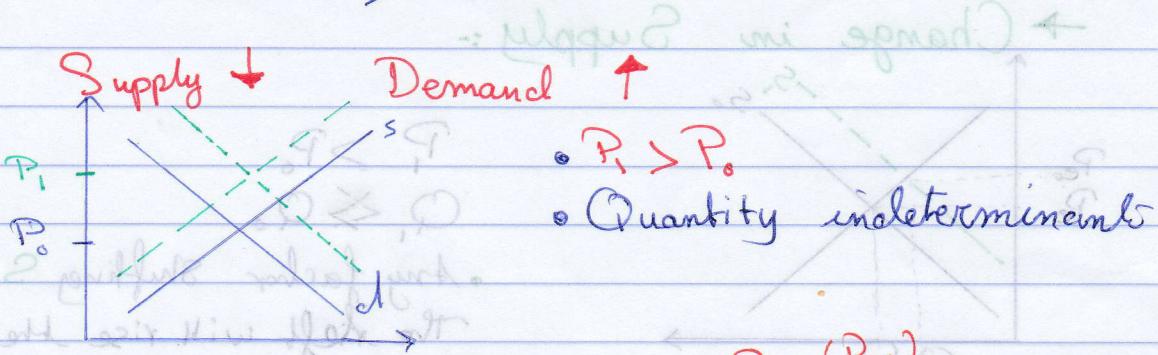
Supply ↑ Demand ↓



$$P_1 < P_0$$

• Quantity indeterminate

Supply ↓ Demand ↑



$$P_1 > P_0$$

• Quantity indeterminate

$P_x$  (Pex)

- Government Revenue =  $(P_{\text{consumer}} - P_{\text{producer}}) Q$
- Total Revenue =  $P Q$

$P_x - t$

of 22 points total price  
will now drop off

# Engineering Economy:

## Three Basic Concepts:

1) Resources → Free: Zero-price  
→ Economic: Non-Zero price

limited / can be put in different Production activities

• What we call:-

Factors of Production:-

- 1 - labour → Payments (wages and salaries),
- 2 - land → Rent.
- 3 - Capital → Int.
- 4 - Entrepreneur → Profit.

2) Alternative Uses:-

Mutually exclusive: Meaning you choose one of the Alternatives only.

→ opportunity Cost: القيمة المضارة

The value of the next Best alternative that is Given up.

You should choose the best Alternative.

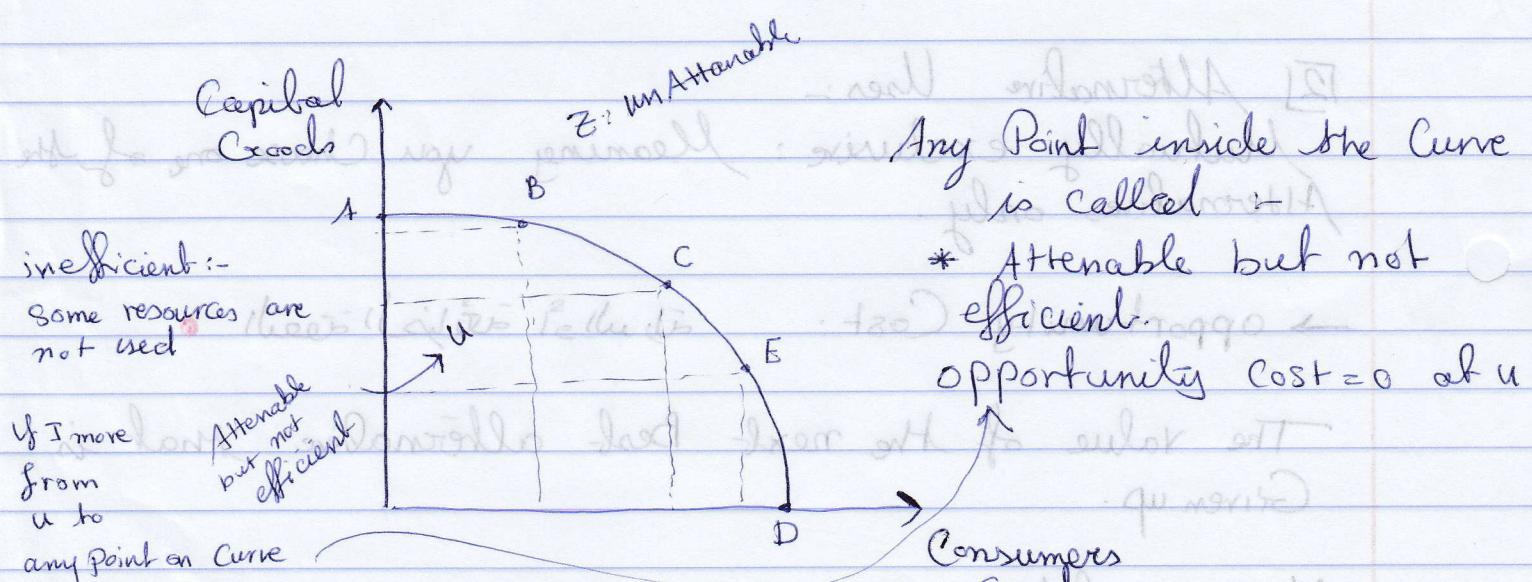
3) Human Needs (wants)

Properties:-

- a) They vary among individuals.
- b) ~ ~ over time for the same individual.
- c) They are unlimited.

Def: Production Possibility Curve: P.P.C or P.P.F

Combinations	Consumers Goods	Capital Goods
A	0	15
B	2	14
C	4	12
D	12	0



1. A: All resources are used to produce Capital Goods

2. D: All resources are used to produce Consumer Goods

Any Point on the Curve (down) is called Inefficient

\* Attainable and efficient

## • Engineering Economy: For decision making. lec(2)

### \* Production possibility frontiers:

- Def: Check different combinations of goods and services producers can produce efficiently by reallocating the available Resources and technologies

→ Shifting happens in our Curve if we change Resources or Technology or both.

### Economic Theory

#### MicroEconomic Theory

- individual Behaviour
- Employment.
- we are interested in it.

#### MacroEconomic Theory

- We deal with inputs as a whole → GDP
- انتشار في الأسعار

CPI: Consumer Price index

الناتج المحلي الإجمالي

نوعية الحياة المعيشية

الارتفاعات المعيشية

- labour force

### Micro Economics

- Demand and Supply
- Product Market

Demand side supply side

products (Goods) tangible

Services

→ Def: Shows different Various Quantities of

a product. Consumers are willing and able to buy at different Alternative Prices during a Specific Period of time holding other factors constant.

## Main determinants of Quantity of a product:-

→ various Quantities --- prices

↳ ~~Price, quantity and quality.~~

~~Divers have cheap to withdraw traffic costs etc.~~

⚠ Note: Prices of the Goods is a Main determinant but it's not the Only One.

Q<sub>dx</sub> → Quantity Demand of goods × unit price

1- Price of Goods P<sub>x</sub>

Related

Substitute Goods

2- Price of Goods P<sub>y</sub>

Product

Complement Goods

3- Number of Consumers N<sub>c</sub>

4- Consumer Taste T

5- Consumer income I

6- Consumer Expectation E

## General Demand Function:-

$$Q_{dx} = f(P, \bar{P}_r, \bar{N}_c, \bar{T}, \bar{I}, \bar{E}, \dots)$$

## Simple Demand Function:-

$$\frac{dQ}{dx} = f(P_x) \quad \text{Given other factors are constant}$$

\* Now,

$$P_x \rightarrow Q_{dx} \quad \frac{dQ_{dx}}{dP_x} < 0$$

Let P → Q<sub>dx</sub>  
Substitute

$\Delta Q_{cl \text{ per }} > 0$

$\Delta P_{\text{scale}} < 0$

(Ex.) at Scale

# Engineering Function Economic

## General Demand function:

lec 3

$$Q_{dx} = f(P_x, I, T, P_y, N_c, E, \dots)$$

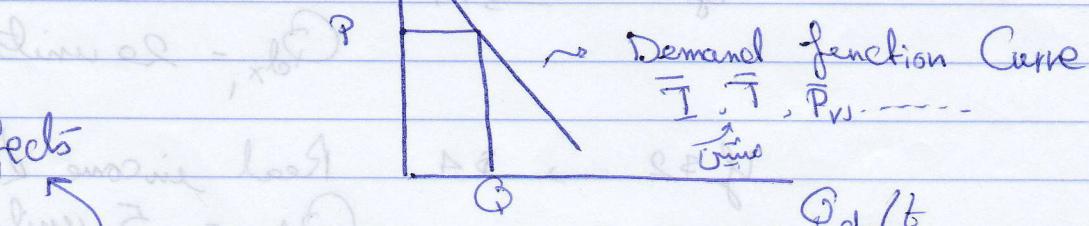
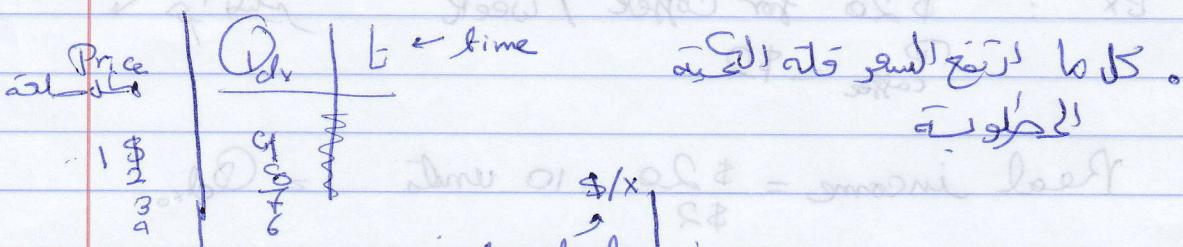
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simple Demand function

$$Q_{dx} = f(P_x) \text{ Given other factors}$$

$$Q_{dx} = \alpha - bP_x$$

$$Q_{dx} = 10 - P_x \text{ equation}$$



Two effects

- law of demand : The higher the Price the lower the Quantity

$$\begin{array}{l} P_x \uparrow \\ Q_{dx} \downarrow \end{array}$$

$$\begin{array}{l} P_x \downarrow \\ Q_{dx} \uparrow \end{array}$$

① Income effect

② Substitution effect :

$$Q_{dx} \rightarrow P_x \quad | \quad \bar{P}_x \text{ per unit price} \rightarrow \text{unit cost}$$

- ② Substitution effect  $\rightarrow \Delta Q_{dx} \rightarrow$  Changes in the Relative prices  $\rightarrow \frac{P_x}{P_y} \rightarrow$  Given  $\bar{P}_y$

Evel

Ex :  $\frac{P_{x_0}}{P_{y_0}} = \frac{\$10}{\$10} \Rightarrow Q_{dx_0} = 100$  (Normal)

$\Delta P_x \Rightarrow \frac{P_{x_1}}{P_{y_0}} = \frac{\$8}{\$10} \Rightarrow Q_{dx_1}$  (Movement along demand curve)

$\Delta P_x \Rightarrow \frac{P_{x_2}}{P_{y_0}} = \frac{\$12}{\$10} \Rightarrow Q_{dx_2}$  (Shift in demand curve)

① Income effect → Real income = Nominal income

Ex : \$20 for coffee / week

$P_{\text{coffee}} = \$2$

Real income =  $\frac{\$20}{\$2} = 10 \text{ units} = Q_{dx_0}$

If  $\$2 \rightarrow \$1$  Real income ↑

$Q_{dx_1} = 20 \text{ units}$

If  $\$2 \rightarrow \$4$  Real income ↓

$Q_{dx_2} = 5 \text{ units}$

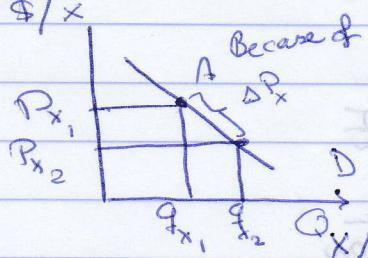
\* The two effects are negative and they describe the Demand Curve.

Distinguish Changes in  $Q_{dx}$  (Quantity demanding) or Movement Between along Demand Curve and Shift in Demand or Changes in Demand.

all we need ← shift in demand curve

$\bar{P}_{\text{new}} \leftarrow x \bar{P} \leftarrow \text{original price}$

- If  $P_x$  only changed

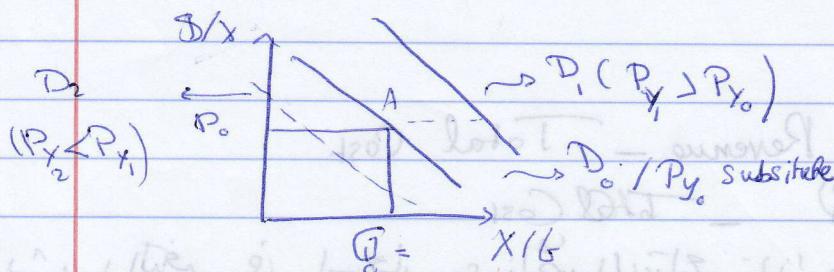
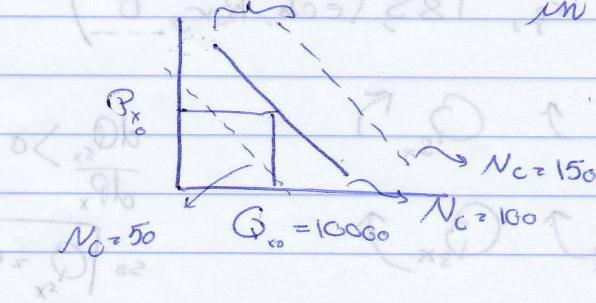


There will be a change in Quantity demanded meaning: A Movement on the Demand Curve.

(Remember !: Other factors are Constant)

or any other factor

- If The Demand Shifted then there will be a Shift in the Demand Curve.



- Supply Curve Shows the various Quantity of a Product Producers (firms) are willing and able to produce / sell at different alternative Prices during a Specific period of time holding other Factors Constant.

→ Supply ( $P_x$ ) is a main Determinant to the Quantity Supplied.

- the Factors Given / holding them Constant

↳ ① Number of firms (producers)  $N_f$

② Price of factors of Production  $P_f$

- (3) Taxes and Subsidies T & S
- (4) Technology Tech
- (5) Price
- (6) Expectations of Producers. E

## General Supply Function

$Q_{S_x}$ : Quantity Supplied of Good X

$$Q_{S_x} = f(P_x, N_f, \bar{P}_f, \bar{T} & S, \bar{Tech}, \bar{R}_r, \bar{E})$$

$$P_x \rightarrow Q_{S_x} \Rightarrow P_x \uparrow Q_{S_x} \uparrow$$

$$N_f \rightarrow Q_{S_x} \Rightarrow N_f \uparrow Q_{S_x} \uparrow$$

$$\frac{dQ_{S_x}}{dP_x} > 0$$

$$\text{so } Q_{S_x} = \alpha + \beta P_x$$

$P_f$ :

$$\begin{aligned} \text{profit} \rightarrow \Pi &= \text{Total Revenue} - \text{Total Cost} \\ &= P \cdot Q - \text{Total Cost} \end{aligned}$$

$\hookrightarrow$  Total Profit = Total Revenue - Total Cost  
Profit = Sales - Total Cost

Taxes  $\rightarrow$  negative

Subsidies  $\rightarrow$  positive

Technology: Positive (if improvement)

Expectation: - (Prices  $\uparrow \rightarrow Q \uparrow$ )

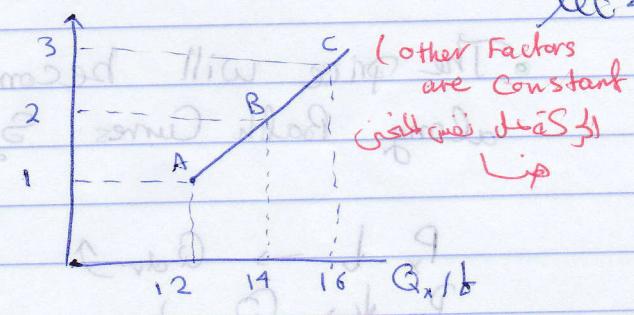
(Prices  $\downarrow \rightarrow Q \downarrow$ )

$$Q_x = f(P_x) \quad (\text{Given other Factors})$$

Price

	$Q_x/16$
1	12
2	14
3	16
4	18

Price \$/x



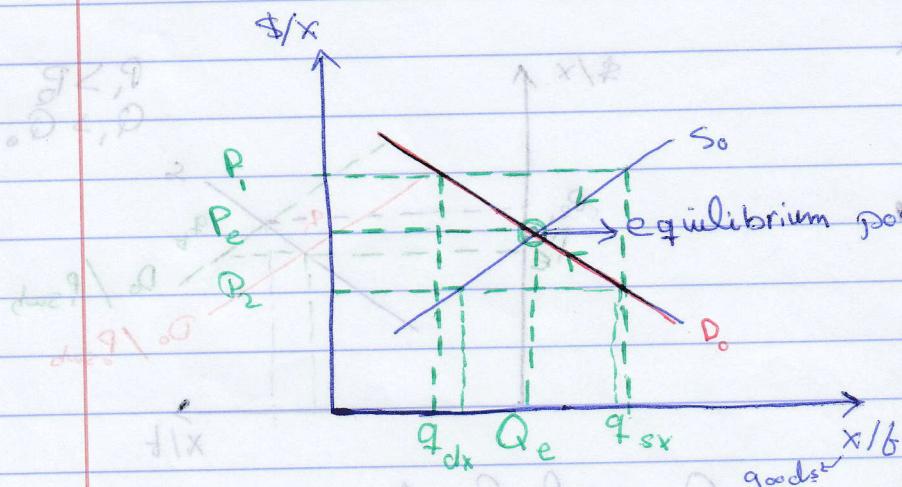
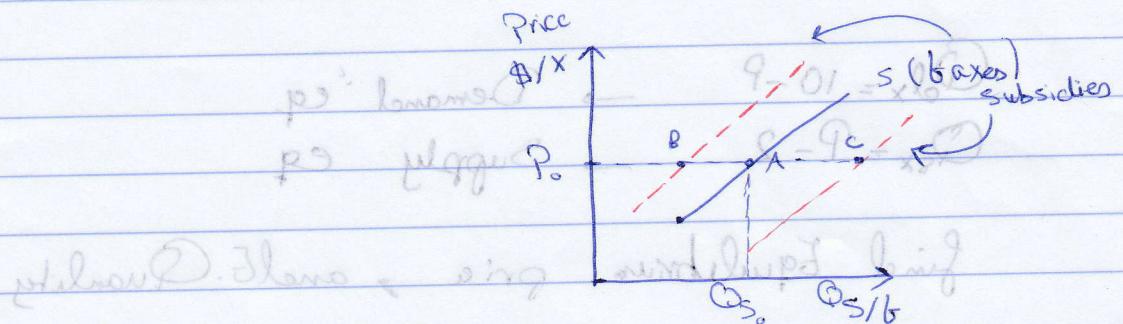
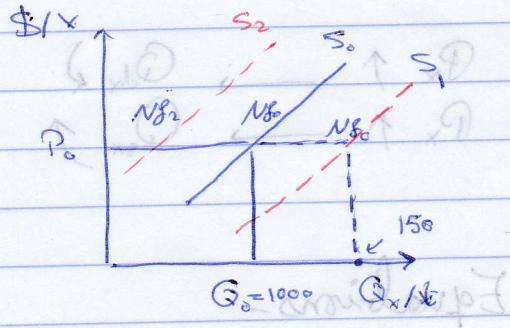
نفترض أن العوامل فقط (غير المحسنة) ✓

- Movement along Supply Curve or = changes in Quantities Supplied and Shift of Supply Curves or = changes in Supply Curve
- غير المحسنة عوامل (non factors)

$$Nf_0 = 100 \text{ firms}$$

$$Nf_1 = 150 \sim$$

$$Nf_2 = 50$$



Equilibrium Condition

$$Q_{dx} = Q_{sx} \text{ is at}$$

Equilibrium position

↓ Equilibrium price  $P_e$ ↓ Quantity  $Q_e$ 

- At  $P_1 \rightarrow Q_{sx} > Q_{dx}$  Surplus / excess supply

D

+ red

- The price will become less so there's a movement along both Curves  $S$  and  $D$ .

$$\begin{aligned} P_x \downarrow &\rightarrow Q_{dx} \uparrow \\ P_x \downarrow &\rightarrow Q_{sx} \downarrow \end{aligned}$$

*(Price fall in both cases)*

- At  $P_2 \rightarrow Q_{dx} > Q_{sx}$  *excessive demand demand* or shortage

- The Price will rise so there is a  $\rightarrow \rightarrow \rightarrow \rightarrow$

$$\begin{aligned} P_x \uparrow &\rightarrow Q_{dx} \downarrow \\ P_x \uparrow &\rightarrow Q_{sx} \uparrow \end{aligned}$$

### Equations -

$$Q_{dx} = 10 - P \rightarrow \text{Demand eq}$$

$$Q_{sx} = P - 2 \rightarrow \text{Supply eq}$$

find Equilibrium price and Q. Quantity

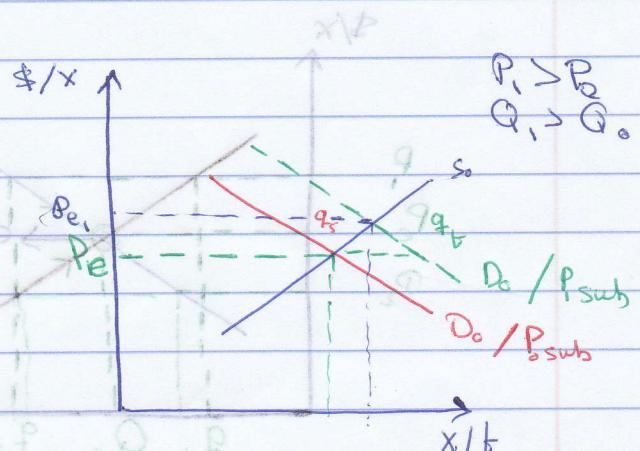
$$Q_{dx} = Q_{sx}$$

$$10 - P_e = P - 2$$

$$-2P_e = -12$$

$$P_e = 6$$

$$Q_e = 4 \text{ units}$$



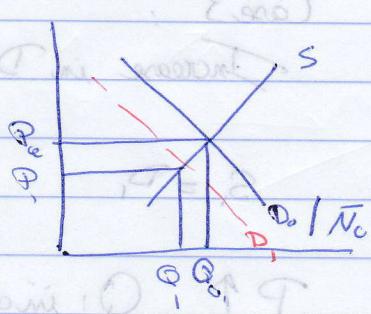
- any factor would shift Demand to the right which makes price increase

General Conclusion: shortage or excess of demand always makes prices increase

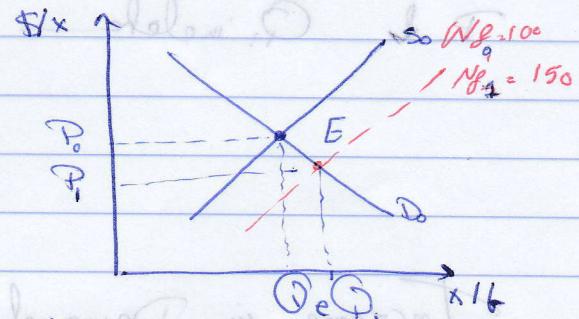
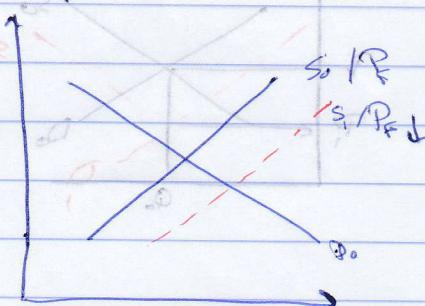
$\sim$  of Supply  $\sim$   $\sim$   
 $\sim$  decrease  $\sim$   $\sim$

\*  $D_1 = S_0$  is the new equilibrium  
Case

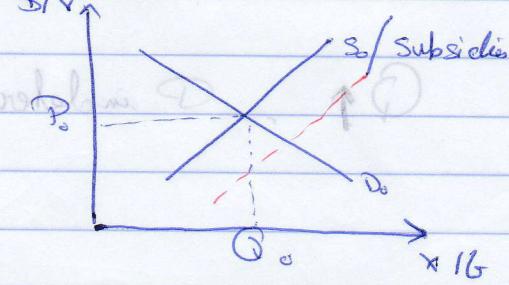
Negative Change  $\rightarrow P$  and  $Q \downarrow$



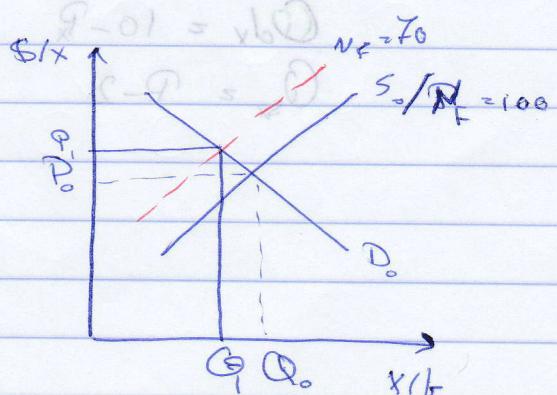
Conclusion 2: Any Factor shifts the demand to the left and does not affect the Supply will lower the eq. price and Quantity.



Conclusion 3: Any shift in supply curve to the right (Dem. constant)  
and increase eq. Quantity.



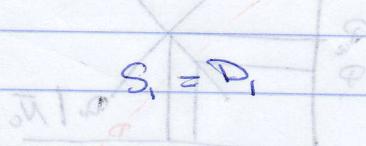
Conclusion 4: Shift in demand (Dem. Cons)  
eq. price ↑ and lower eq. Quantity



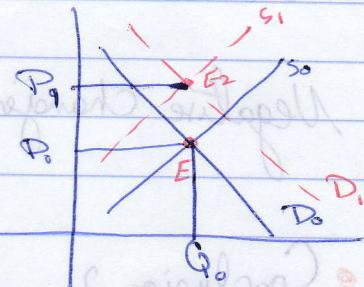
(d) Cool

Case 3:

Increase in Demand, Decrease in Supply



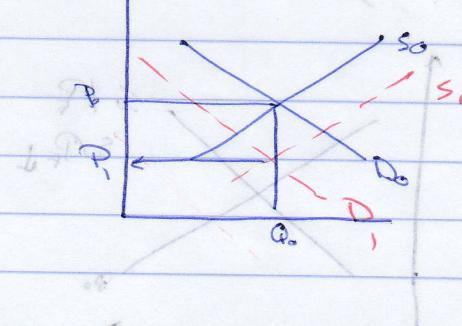
$P \uparrow$ ,  $Q_1$  indeterminate



which needs only after what part (minimum).

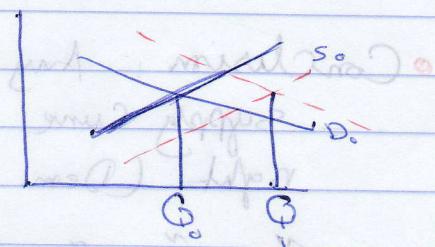
What happens at traffic jams etc. long etc.

Increase in Supply, decrease in demand



Increase in Demand, Increase in Supply

$Q_1 \uparrow$ ,  $P$  indeterminate



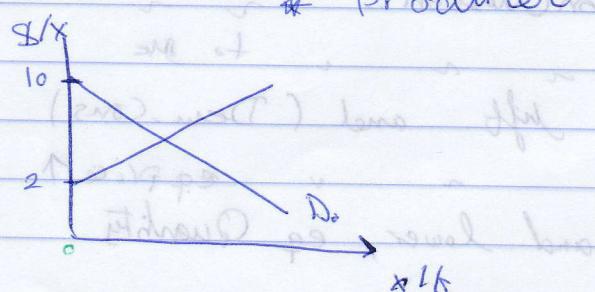
decrease in

$$Q_{dx} = 10 - P$$

$$Q_{sx} = P - 2$$

\* Consumer Surplus

\* Producer Surplus



$P > \text{avg}$   $\rightarrow G$

$P < \text{avg}$   $\rightarrow Q$

## • Elasticity of Demand and Supply :-

- 1 Price elasticity of demand
  - Point elasticity
  - midpoint elasticity
- 2 Cross-price elasticity of demand
- 3 Income elasticity of demand

①  $E_p = \frac{\% \text{ Change in the dependent}}{\% \text{ Change in the independent}}$

$\% \text{ Change in } Q = Q_d - Q_{d_0} / Q_{d_0}$

$\% \text{ Change in } P_x = P_x - P_{x_0} / P_{x_0}$

$$E_p = \frac{\Delta Q_{dx}}{\Delta P_x} \cdot \frac{P_{x_0}}{Q_{d_0}}$$

Point (ignore the sign)

$$E_{p\text{mid}} = \frac{\Delta Q_{dx}}{\Delta P_x} \cdot \frac{(P_{x_1} + P_{x_0})/2}{(Q_{x_1} + Q_{x_0})/2}$$

→  $E_p$  has Three possibilities :-

$$E_p > 1$$

elastic

$$\therefore Q > \frac{1}{E_p} P$$

more sens

$$E_p < 1$$

inelastic

$$\therefore Q < \frac{1}{E_p} P$$

less sens

$$E_p = 1$$

unitary

$$\therefore Q = \frac{1}{E_p} P$$

$$E_p = \frac{\text{Marginal function}}{\text{Average function}} = M.P$$

$$A.P = Q_a/P_a$$

## Determinants of Price elasticity of demand:-

① The availability of Substitutes Goods  
Make it More elastic

② Proportion of income

income  $\leftarrow$  inelastic  $\rightarrow$  elastic

③ Type of products

SPLAT

Luxuries Vs Necessities  
Elastic Inelastic

④ Time :-

Short period, inelastic Time that takes people to notice the change  
long period : elastic People needs time to find substitutes

⑤ habits (Addiction) inelastic / rigidly

## ② Cross Elasticity of Demand - Measures sensitivity of

$$E_{\text{cross}} = \frac{\% \Delta Q_{\text{dx}}}{\% \Delta P_y}$$

$E_{\text{cross}}$  due to change in the price of other Goods (y)

$E_c > 0$ :  $P_y \uparrow Q_x \uparrow$  Two Goods are Substitutes  
 $P_y \downarrow Q_x \downarrow$

$E_c < 0$ :  $P_y \uparrow Q_x \downarrow$  Two Goods are Complements  
 $P_y \downarrow Q_x \uparrow$

$E_c = 0$ : No Relation between Goods

### ③ Income Elasticity

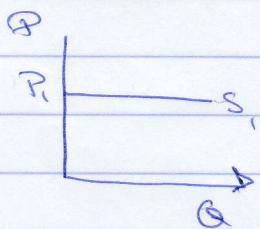
$$E_{\text{inc}} = \frac{\% \Delta Q_{dx}}{\% \Delta I}$$

$E_I > 0$      $I \uparrow \rightarrow Q_{dx} \uparrow$     Normal Goods / Superior  
 $I \downarrow \rightarrow Q_{dx} \downarrow$

$E_I < 0$      $I \uparrow \rightarrow Q_{dx} \downarrow$     Inferior Goods  
 $I \downarrow \rightarrow Q_{dx} \uparrow$

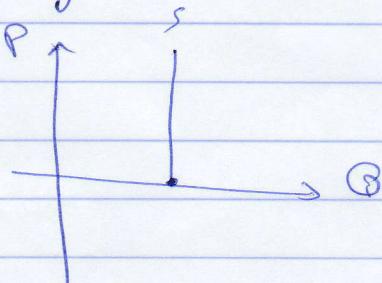
#### Note

- Elastic and inelastic supply curve



No change in price

Supply is constant (fixed)



- Total Revenue Test

$$TR = P \times Q$$

- ① demand is elastic

$$E_p > 1$$

$$E_d = \infty$$

$P \downarrow Q \uparrow TR \uparrow$

$P \uparrow Q \downarrow TR \downarrow$

$$E_p = 1$$

No change in TR



$$E_d > 1$$

$$E_d < 1$$

- ② demand is inelastic

$P \uparrow TR \uparrow$

It doesn't

$P \downarrow TR \downarrow$

have effect  
in Quantity

Price elasticity of Supply  $E_s = \frac{\frac{1}{2} \frac{\Delta Q_s}{Q_1 + Q_2}}{\frac{1}{2} \frac{\Delta P}{P_2 - P_1}} = \frac{\frac{Q_2 - Q_1}{(Q_2 + Q_1)/2}}{\frac{P_2 - P_1}{(P_2 + P_1)/2}}$

$$\begin{array}{lll} Q_1 & Q_s \uparrow & (+) E_s > 0 \\ P_1 & Q_s \uparrow & \end{array}$$

$$\frac{+bD1X}{TAX} = \frac{2(Q_2 - Q_1)}{Q_2 + Q_1} + \frac{2(P_2 - P_1)}{(P_2 + P_1)}$$

$E_s < 1$ : short period, perfectly inelastic  $\leftarrow \uparrow$  time is the main determinant

$E_s > 1$ : long period, more elastic  $\leftarrow \downarrow$  Determinant

$E_{Ps} = 0$ : The Good is fixed in Supply (perfectly inelastic)  $>$  (short period)

$E_{Ps} = 1$ : unit elastic

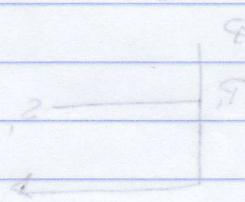
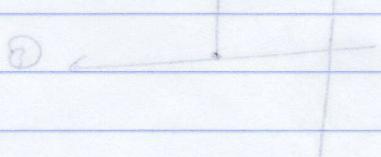
$$\frac{2(Q_2 - Q_1)(P_2 + P_1)}{2(Q_2 + Q_1)(P_2 - P_1)}$$

$$\frac{2(Q_2 - Q_1)(P_2 - P_1)}{2(Q_2 + Q_1)(P_2 + P_1)}$$

still

and change situation long period

(long) instead of change



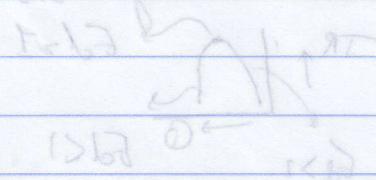
fixed quantity later.

$$Q \times P = RT$$

$$1 = Q \cdot P$$

$K_p$  circle a branch

ATM in Germany



$\uparrow AT \uparrow Q \uparrow P$

$\uparrow ATM \uparrow Q \uparrow P$

fixed

$\uparrow ATM \uparrow P$

ATM not  
in Germany

$\uparrow ATM \uparrow P$

## • Production function :-

- Types,
  - ① Simple prod. function
  - ② Production function with  $f$  variables

$$\rightarrow Q = f(V_1, V_2, V_3, \dots, V_n)$$

↑ inputs  
 Fixed      Variable  
 Capital (K)      Labor (L)

$$\text{so } Q = f(L, K) \quad (\text{Non-Substitution of input})$$

No substitution between factors of inputs

$$\begin{matrix} L \uparrow & K \downarrow \\ L \downarrow & K \uparrow \end{matrix}$$

Firm Optimization

$Q$  Given  $\rightarrow$  Cost Min  
 $\text{Cost Given} \rightarrow Q$  Max

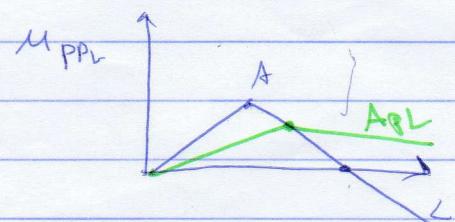
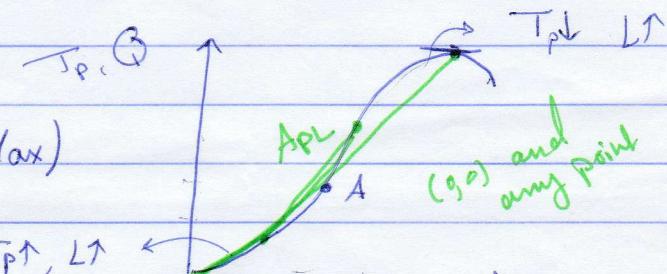
$$\begin{aligned} \Pi &= TR - TC \\ &= P \times Q - TC \end{aligned}$$

- ① S.P.F :-
- T.P. b <sup>physical</sup> (single variable input)
  - $MPP_L$ , Marginal products
  - $AP_L$ : Average production of Labor
  - Relation between  $AP, MPP, TP$
  - Marginal Revenue of Product  $\leq$  Total Product

- A is when slope begins to come down ( $MPP_L$  is Max)

~~Diagram~~  $MPP_L = \text{slope } (TP \text{ vs } L) = \frac{\Delta Q}{\Delta L}$

$$AP_L = \frac{Q}{L}$$



## The Relationship Between $A_{PL}$ and $M_{PP_L}$

$M_{PP_L} > A_{PL}$  when  $A_{PL}$  is increasing

$M_{PP_L} = A_{PL}$  when  $A_{PL}$  is Max

$M_{PP_L} < A_{PL}$  ~  $A_{PL}$  is decreasing

### ② 2. V.P.F

L, K are substitutable

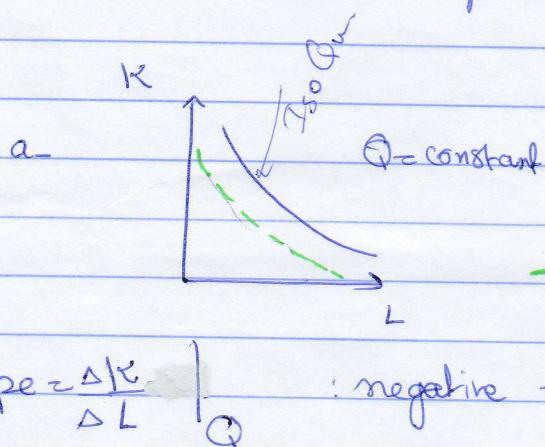
a - Iso Quant Curve = Production Indifferent Curve

Slope - MRTS : Marginal Rate of Technical Substitution

- Budget line (constraint) = Iso Cost line

- least Costly input Combination

- law of Returns to scale



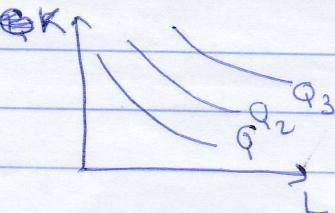
Different values of  $R_s, L, K$

Gives us same value of  $Q$

→ Shifting (in case of Technical improvement)  
Same output but less input  $Q = 10\sqrt{LK}$

$(\downarrow \rightarrow)$   $Q = 20\sqrt{LK}$

→ Iso-Quant Map =



• If the Curve is in L shape =

L, K are Complement

(Fixed proportions production)

Slope = 0

• ~ ~ ~ line

L, K are Substitutional

Slope is constant

## • Cost Function

### Short Run Costs -

1- TFC

2- TVC

3- TC

4 AFC

5 AVC

6 ATC

$$G = AFC + AVC$$

7 Marginal Cost =  $\frac{TC}{dQ}$  if all remains

$$\frac{M_C = dTC}{dQ} = \frac{(dTC = 0)}{dQ}$$

constant

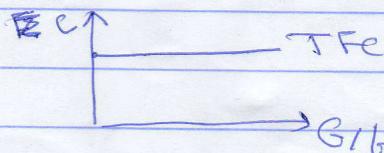
$$= \frac{W}{MPPL}$$

① Cost of fixed inputs / can't be Returned

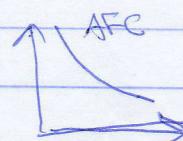
② They don't vary

- FC > 0 even if output = 0

= investment Cost = Capital Cost

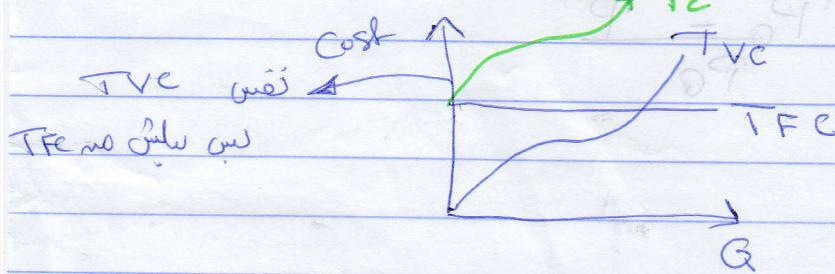


$$\bar{AFC} = \frac{TC}{Q}$$

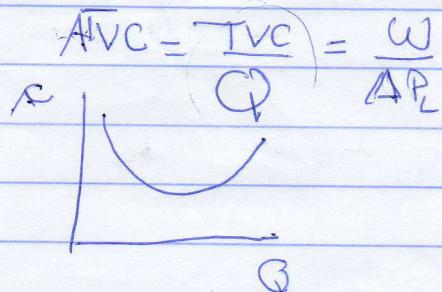


② TVC - Total opport. Cost =  $\frac{\Delta TC}{\Delta Q}$   $TC - TFC = W \cdot L$

+ ⑤ Annual expenditure / annual disperment Cost

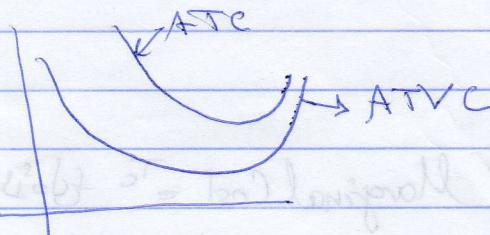


$\text{if } \beta = 0 \Rightarrow TVC = 0$



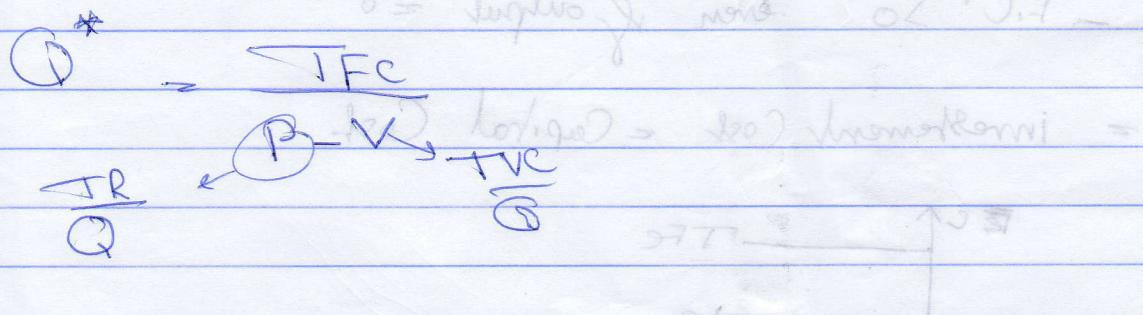
$$③ TC = TVC + TFC$$

nut -> (Co).



→ ~~AVC~~ min (Cost)

$$Q^* = \frac{FC}{TR - TVC} \rightarrow \text{productive capacity}$$

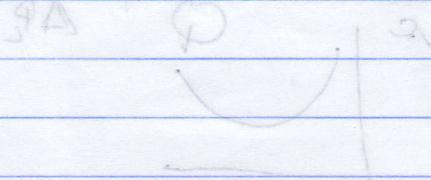


$$P^* \Rightarrow TR - TC$$

$$P^* = AFC + TVC$$

$$\rightarrow \text{Price Safety margin} = \frac{P_Q - P^*}{P_Q}$$

$$W = AVC - ATC$$



$$W = ATC - AVC$$

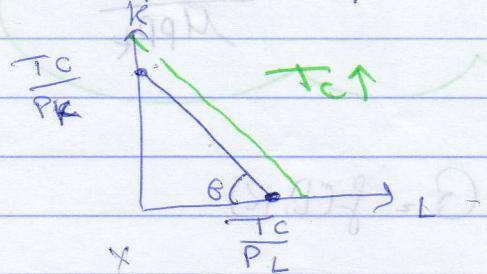
$$ATC = AFC + AVC$$

## Iso Cost line/Curve

Dont work

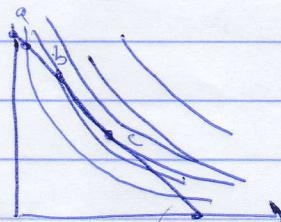
$$TC = P_L \times L + P_K \times K \quad (\text{firm's expenditure equation})$$

- ↳  $TC \rightarrow$  on  $K$  :  $O A$  on curve
- ↳  $TC \rightarrow$  on  $L$  :  $O B$  on curve



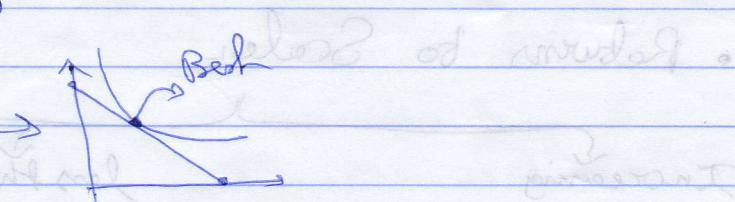
$$\text{Slope} = \tan \theta = \frac{OA}{OB} = \frac{\frac{TC}{P_K}}{\frac{TC}{P_L}} = \frac{P_L}{P_K} = MRTS$$

→ Firm optimization: Iso line + Iso-Cost Map



a, b, c are mutually exclusive  
you can choose one / c is the best

$$MRTS = \frac{P_L}{P_K}$$



Meaning

$$\left[ \frac{P_L}{P_K} = \frac{\Delta K}{\Delta L} \right] \quad \text{firm optimization}$$

## • Prove That

$$MRT_S = -\frac{MPP_L}{MPP_K}$$

→  $Q = f(L, K)$

constant  $\frac{dQ}{dK} + \frac{dQ}{dL} = 0$

$$0 = MPP_K \Delta K + MPP_L \Delta L$$

$$\frac{MPP_L}{MPP_K} = -\frac{\Delta K}{\Delta L} = MRT_S$$

## • Returns to Scale

Increasing  
Return

Input double  
outputs more than double

(Economies)

less than  
double (Decreasing)

Inputs double  
outputs less than  
double

(diseconomies)

exactly  
in double (constant)

Inputs double  
outputs double

