Time - Money Relationship

Interest: الفائدة

- Interest
  - Simple interest = (principal) x (number of periods) x (interest rate)
  - Compound interest: في هذه الحالة يضرب كل مبلغ تم الحصول عليه في نهاية كل عام بـ interest rate

$$= \frac{\text{Payment}}{\text{Principal}} \leftarrow \text{قيمة المدفوعة}$$

- Terminlogy & symbols

P: amount of money at time = 0

F: " " " " time = N (future)

A: Annual worth / consecutive, equal end of period amounts of money.

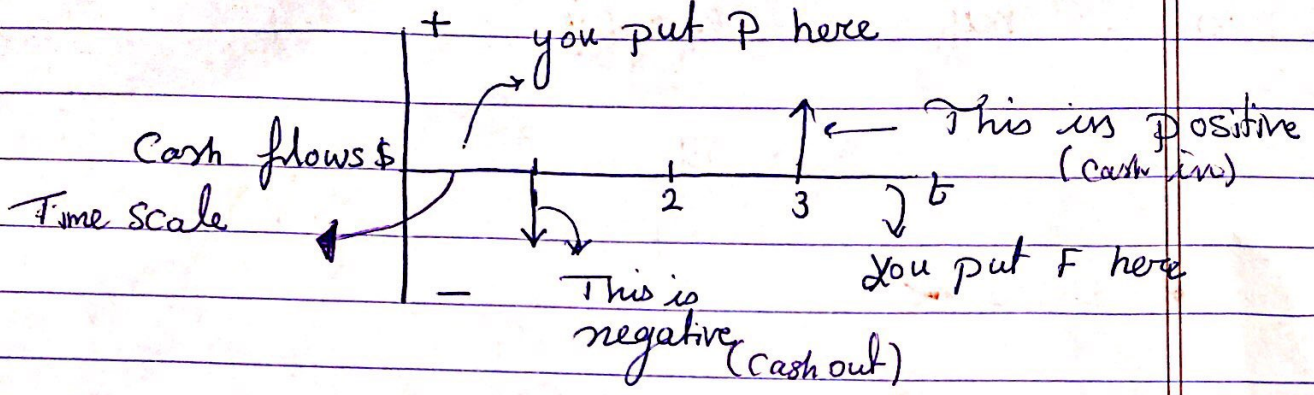
n: period (years, months, days)

i: interest rate (percent)

t: time

- Cash flow Diagram:-

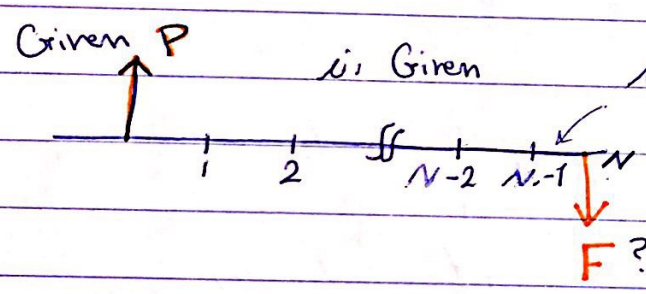
Net Cash flow = Cash inflows − Cash outflows



Economic Analysis using P, F, A, i, N :- (discrete compounding)

① Find F Giving P : for a period N; $F = P(1+i)^N$

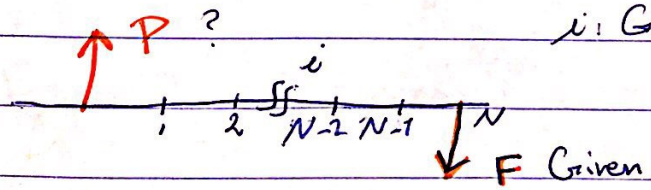
$F = P(F/P, i, N)$ → Single payment compound Factor



Single payment present worth

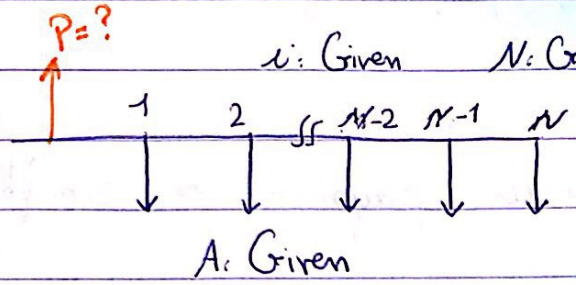
② Find P Given F: for a period N; $P = \frac{F}{(1+i)^N}$

$P = F(P/F, i, N)$



[3] Find P Given A : $P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$

$P = A(P/A, i, N)$

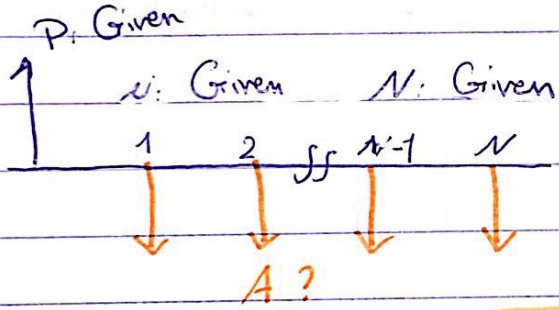


↳ Uniform series present worth factor

[4] Find A Given P :- $A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$

$A = P(A/P, i, N)$

↳ Capital Recovery



↳ Sinking Fund factor

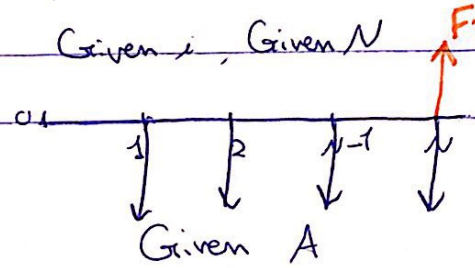
[5] Find A Given F :- $A = F\left[\frac{i}{(1+i)^N - 1}\right]$

$A = F(A/F, i, N)$

↳ Uniform series Compound amount factor

[6] Find F Given A :- $F = A\left[\frac{(1+i)^N - 1}{i}\right]$

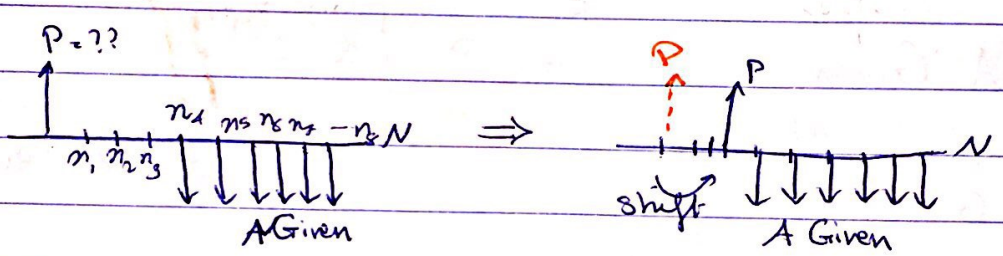
$F = A(F/A, i, N)$



Note : A تتطلق جزئين

interest — Repayment of part of loan

- Deffereed uniform series payment : or shifted payment-



Methods to calculate equivalent present worth P

① ~~F/A factor :- F = A (F/A, i, N) and then P = F (P/F, i, N)~~ ($n_8 - n_1$ ↓ The whole period n_8)

② P/A factor :- P is the future of P_0

$P_8 = A(P/A, i, N)$ ← No. of The whole period only : $n_8 - n_3$

$P_0 = P\ (P/F, i, N)$ ← $n_3 - n_1$

M.A.R.R : hurdle rate

- Economic Methods for evaluating any proposed projects (private projects)

[1] Annual Worth Method:- M.A.R.R known

Net Annual cost flows = Annual cost inflows − Annual Cost outflo
N . A . C . F

$= R - D - CR = P(A/P, i, N) - F(A/F, i, N)$ ↳ Salvage val

R: Annual Revenue / savings ; D: Expenses ; CR: investement Cost

If N.A.C.F (N.A.worth) = 0 : The project has no profit and has no loss

If N.A.C.F > 0 : " " " profit

If N.A.C.F < 0 : " " losses (Not recomoneded)

[2] present Worth Method:-

N. P. W

Net present Worth = Present Worth Cashin flows
− Present Worth Cashout flows.

If N.P.W = 0 : The project has no profit

If N.P.W > 0 : " " " profit

If N.P.W < 0 : " " loses (Not recommended)

$N.P.W = -P + (R-D)(P/A, i, N) + F(P/F, i, N)$

[3] The Future Worth Method

Net Future Worth of cash flows = Future Worth of cash in flows − Future Worth of cash out flows

$N.F.W = F + A(F|A, i, N) - P(F|P, i, N)$

$\therefore A(F|A, i, N) \geq 0$: Justified.

[4] The internal Rate of Return Method

investor's method / discounted cash flow method / profitability index

$I.R.R\ (i^*) > M.A.R.R\ (i)$ The project is acceptable

$I.R.R\ (i^*) < M.A.R.R\ (i)$ " " Not "

i^* Makes $N.P.W = 0$, $N.F.W = 0$, $N.A.W = 0$

i, and N are Known

We use Trial and error to calculate N.P.W using different values of i^*

Then we bound The value $i_1^* < i^* < i_2^*$

$$i^* = i_1^* + \frac{N.P.W \text{ at } i_1^*}{(N.P.W \text{ at } i_1^* - N.P.W \text{ at } i_2^*)} (i_2^* - i_1^*)$$

- Remember That: M.A.R.R is known in the first three Methods

Note: - N.F.W = N.P.W (F/P, N, i)
N.A.W = N.P.W (A/P, ---)
N.A.W = N.F.W (A/F, ---)

- CR covers two items:
1- loss in value of the asset
2- interest or invested capital (the M.A.R.R)

- **Problems** Concerning IRR Method:-

1- The Method is based on the Assumption that recovered funds, if not consumed, are re invested at i^* rather than at M.A.R.R

2- Computational intractibity and the occurance of multiple IRRs in some types of problems.

3- It must be carefully applied when there is only one Alternative chosen

- **Advantages**:- It's widespread acceptance by industry where various types of rates of return and ratios are routinely used in making project selections.