Feed Back is used to assure zero difference between input and signal feed back from output.

$$X_3 - X_1 - X_2 = X_4$$

$$e\,G_1(S) - y\,G_3(S) - Ky = X_4$$

$$u - y = e$$

$$(u-y)\,G_1(S) - y\,G_3(S) - Ky = \frac{y}{G_2(S)}$$

$$y = X_4\,G_2(S)$$

$$u\,G_1(S) - y\,G_1(S) - y\,G_3(S) - Ky = \frac{y}{G_2(S)}$$

$$\frac{\cancel{u}\,G_1(S)}{\cancel{u}} = \frac{y}{u}\left(\frac{1}{G_2(S)} + G_1(S) + G_3(S) + K\right)$$

$$\frac{y}{u} = \frac{G_1(S)}{\frac{1}{G_2(S)} + G_1 + G_3 + K}$$

$$\frac{y}{u} = \frac{G_1 G_2}{1 + G_2(G_3 + G_1 + K)}$$

# Rotation Modelling

$$F = m\frac{d^2x}{dt^2} = m\frac{dv}{dt} \longrightarrow v = \frac{1}{m}\int F\,dt$$ (linear Motion)

$$T = I\frac{d^2\theta}{dt^2} = I\frac{d\omega}{dt}$$

inertia

angular velocity

Torque

## Spring System

### Spring Force

$$F = K(x_1 - x_2)$$ (linear Motion)

$$T = K(\theta_1 - \theta_2)$$ (Rotational Motion)

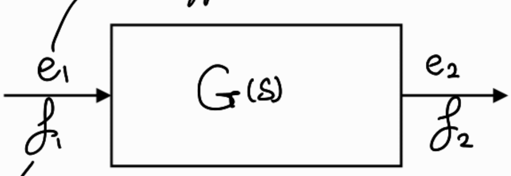
## Damper System

$$F = C(v_1 - v_2)$$ (linear Motion)

$$T = C(\omega_1 - \omega_2)$$ (Rotational Motion)

## Effort and flow

$$\text{Power} = \text{effort} \times \text{flow} = e_1 \times f_1 = e_2 \times f_2$$




effort المسبب

flow التغير

$$f_1 = f_2 = f$$

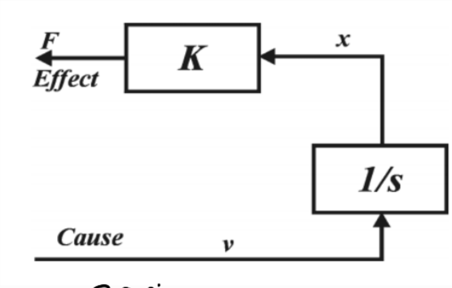
$$\text{Efficiency} = \frac{\text{Power out}}{\text{Power in}}$$

Transfer function $= \frac{e_2}{e_1}$

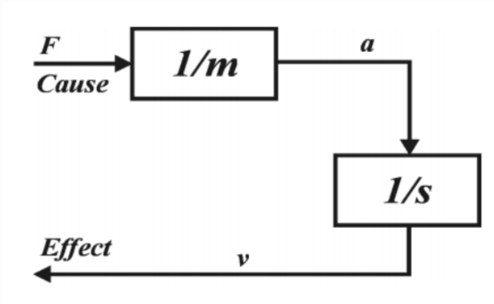
$$\text{Input impedance} = \frac{e_1}{f_1}$$

$$\text{Output impedance} = \frac{e_2}{f_2}$$

There are two cases (Casuality theorem)



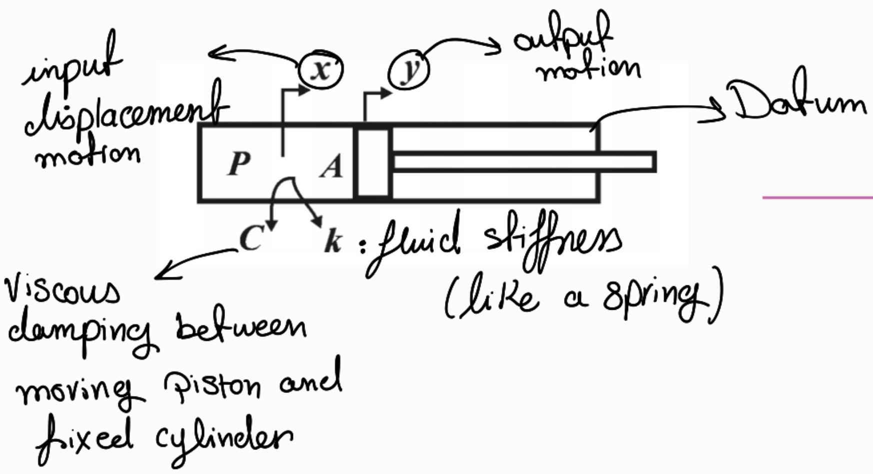

Spring



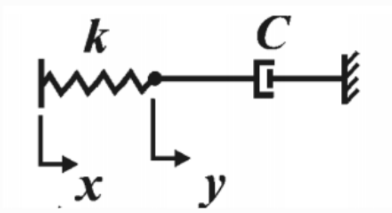

External force on a mass

| General system | Effort $(e)$ | Flow $(f)$ | $\int(f)dt$ | $\int(e)dt$ |
|---|---|---|---|---|
| Mechanical | Force (F) | Velocity (v) | Displacement (x) | Momentum (L) |
| Mechanical | Torque (T) | Angular velocity $(\omega)$ | Angle $(\theta)$ | Angular momentum (H) |
| Electrical | Voltage (V) | Current (i) | Charge (q) | Flux $(\phi)$ |
| Fluid | Pressure (P) | Flow rate (Q) | Volume (V) | Pressure momentum $(\Phi)$ |

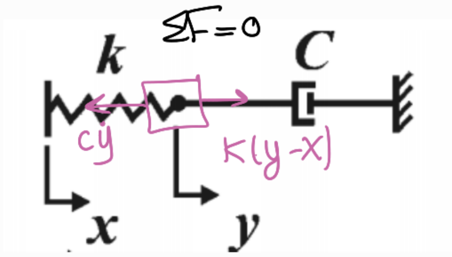
## Modelling of a piston

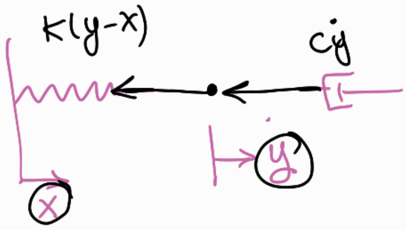



$\Sigma F = 0$



# 2 degrees of freedom System




$$Kx - Ky - C\dot{y} = 0$$

## Analysis in time domain

- time domain
- in terms of t

$$K(x-y) - C\dot{y} = 0$$

$$K(x-y) = c\dot{y}$$

$$K(x-y) = c\frac{dy}{dt}$$

$$\frac{K}{c}\,dt = \frac{1}{(x-y)}\,dy$$

$$\frac{K}{c}t = -\ln(x-y)$$

$$e^{-\frac{K}{c}t} = x-y$$

$$y = x - e^{-\frac{K}{c}t} = x - e^{-\frac{t}{\tau}}$$

$\tau = \frac{c}{K}$ : time constant

## Analysis in frequency domain

- sinusoidal input
- laplace transform

$$K(x-y) = c\dot{y}$$

$$K(x-y) = csy$$

$$Kx - Ky = csy$$

$$Kx = csy + Ky$$

$$Kx = y(cs+K)$$

$$\frac{y}{x} = \frac{K}{cs+K}$$

$s = \omega j$

frequency

$$\frac{Y(s)}{X(s)} = \frac{1}{\frac{c}{K}s + 1}$$

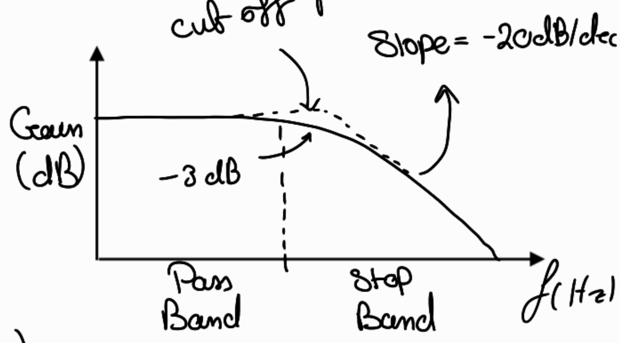
$\tau$

## Bode plot

Decibles: $dB = 20 \log_{10} \frac{A}{B}$

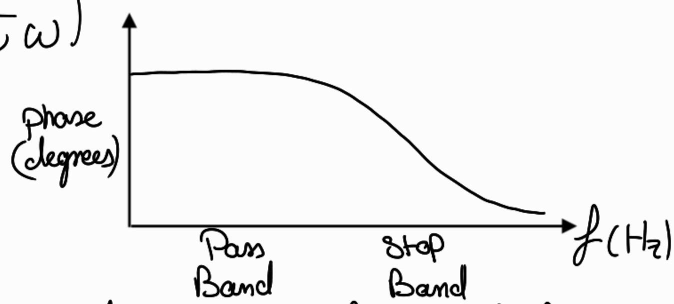
for amplitudes

or 10 for power




Phase angle $\emptyset$: $\emptyset = \tan^{-1}(-\tau\omega)$

in degrees

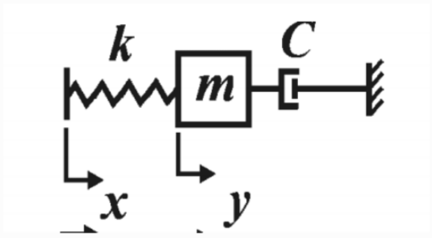



## low pass filter

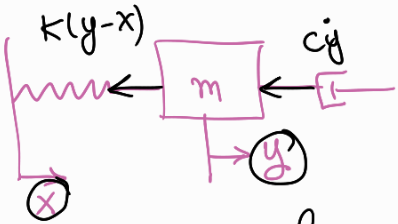
- It passes signals with frequency lower than a certain cut-off frequency

> First Order

- filter gain is K
- Supresses with 20 dB




# 2 degrees of freedom System




$$Kx - Ky - C\dot{y} = \underbrace{m\ddot{y}}_{\text{inertia force}}$$

## Analysis in time domain

- time domain
- in terms of t

$$K(X-Y) - C\dot{y} = m\ddot{y}$$

$$K(X-Y) = C\dot{y} + m\ddot{y}$$

$$K(X-Y) = c\frac{dy}{dt} + m\frac{d^2y}{dt^2}$$

## Analysis in frequency domain

- sinusoidal input
- laplace transform

$$K(X-Y) = C\dot{y} + m\ddot{y}$$

$$K(X-Y) = CsY + ms^2Y$$

$$Kx - Ky = Cs\,y + ms^2y$$

$$Kx = Csy + Ky + ms^2y$$

$$Kx = y(Cs + K + ms^2)$$

$$\frac{y}{x} = \frac{K}{ms^2 + Cs + K}$$

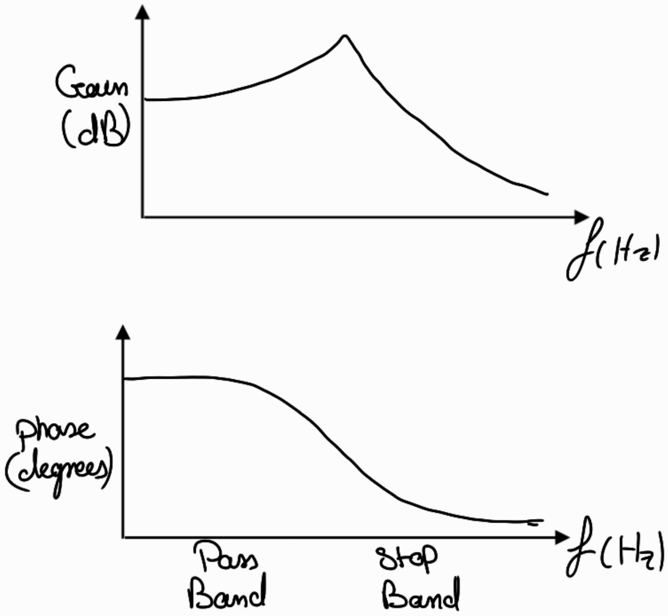
$$\frac{y}{x} = \frac{K/m}{s^2 + \frac{C}{m}s + \frac{K}{m}}$$

$K/m \rightarrow \sqrt{\omega n}$

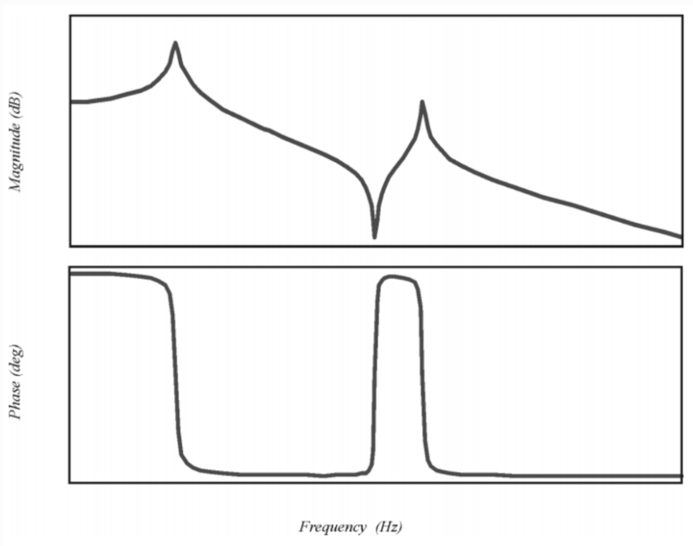
$\frac{C}{m} \rightarrow 2\xi\omega n$

> Second Order

- filter gain is K/m
- Supresses with -40dB



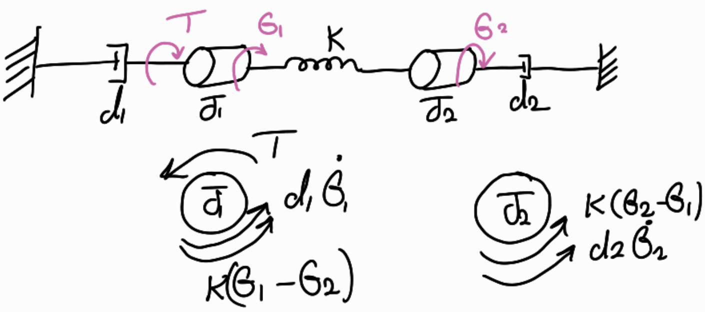

## Systems with two equations




## Modelling of rotating elements

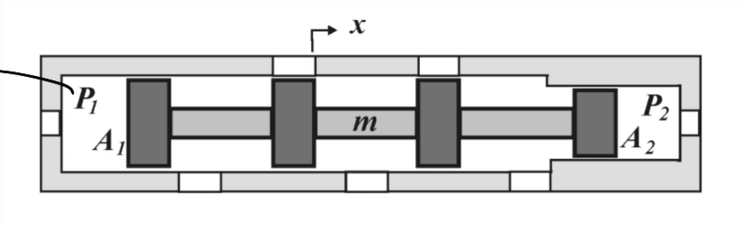
$$T - d_1\dot{\theta}_1 - K(\theta_1 - \theta_2) = J_1\ddot{\theta}_1$$

$$-K(\theta_2 - \theta_1) - d_2\dot{\theta}_2 = J_2\ddot{\theta}_2$$



# Modelling of control valves

low pressure




$$P_1 A_1 - P_2 A_2 = m\ddot{x} + C\dot{x} + Kx$$

$$Q = C_d A \sqrt{\frac{2\Delta P}{\rho}}$$

high pressure difference