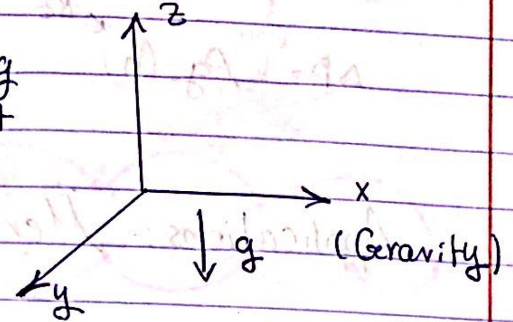
Chapter 2:

Hydrostatic fluid

2.1 Pressure and Pressure Gradient

- In Hydrostatic fluid as long as you're moving in x, y direction, P_z won't Change: (P is changing vertically)

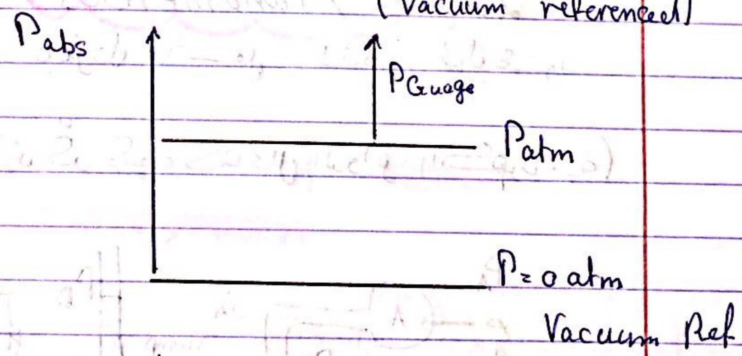


$P_z = P_n + \frac{1}{2} \rho g \, \Delta z$

الضغط متعود
الـثقل

2.2 • P_n: is pressure Change in the horizontal Direction and It equals zero here.

2.3 pressure →
- **Guage pressure** = $P_{abs} - P_{atm}$ (atm referenced)
- **absolute pressure** = $P_{guage} + P_{atm}$ (Vacuum referenced)



As we Knew before: $\frac{dp}{dz} = -\gamma$

γ → specific weight — تغير وزن عمود الـثقل

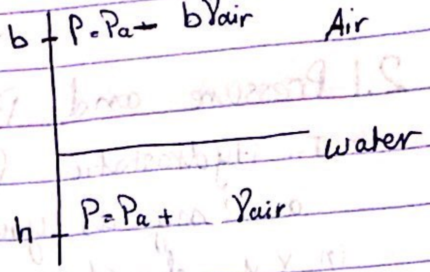
$\frac{dp}{dz}$ → - تغيّر الضغط بالنسبة لـ z

pressure decreases when we move upward

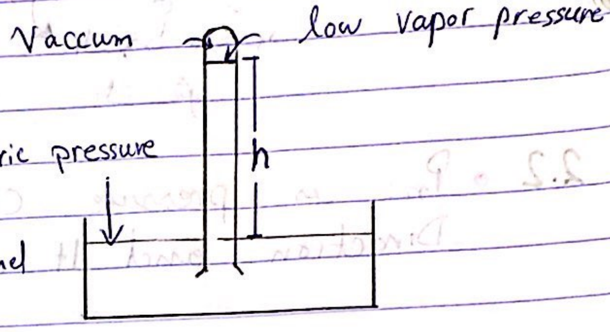
for liquids & from that we conclude that (By solving the differential equation)

$$\Delta z = \frac{\Delta p}{\gamma}$$

$$\Delta p = \rho g h \quad (h \leftarrow \Delta z)$$



Applications: Mercury Barometer



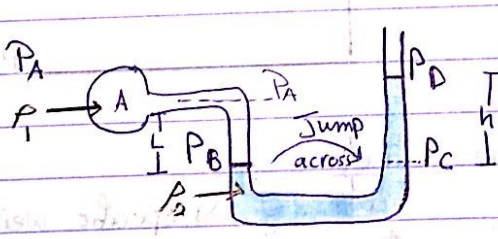
Atmospheric pressure pushes mercury and so mercury rises to height h

2.4 Manometers

تذكر دائماً : الضغط يزداد بالنزول للأسفل لأن طول عمود السائل يزداد.

نعامل الضغط كمتجهات (تذكر كيف كنا نحل الدوائر الكهربائية)

مثال

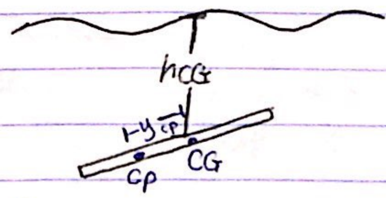


$$P_A + L\rho_1 - h\rho_2 = P_D = P_a$$

Jump across : معناها أن النقطتين على نفس المستوى والضغط متساوي عندهم

2.5: hydrostatic forces on plane surfaces

Force on Body From water and atm is



$$\boxed{F = P_{CG}\, A}$$

where: A is the Area of Body

$$P_{CG} = P_{wat} + P_{atm} = \gamma h_{CG} + P_a$$

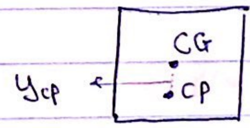
$$y_{cp} = \frac{-\gamma \sin\theta \, I_{xx}}{P_{CG}\, A}$$

$$x_{cp} = -\frac{\gamma \sin\theta \, I_{xy}}{P_{CG}\, A}$$

where: • I_{xx} is moment around x-axis

• θ: Between surface and Body.

⇒ If $I_{xy} = 0$ (Due to symmetry) Then $x_{cp} = 0$ and cp lies under CG Directly



Now: a special case: if $P_{atm} = 0$

Then $P_{CG} = P_{Guage}$ only

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{CG}\, A}$$

$$x_{cp} = \frac{-I_{xy} \sin\theta}{h_{CG}\, A}$$

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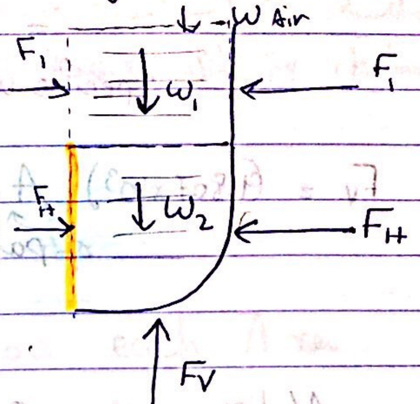
2.6 Hydrostatic forces on curved surfaces

for curved surfaces we always use projection



- The wall will exert ~~an~~ force on the fluid which has two components horizontal and vertical

- There are forces opposing These forces

F_1 exerted by the left side of the fluid column opposes F_H.

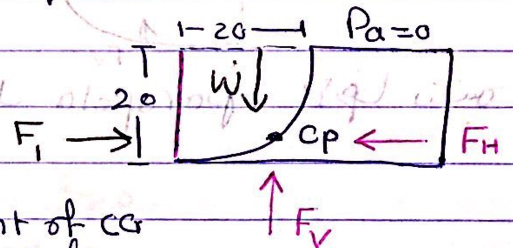
W_1, W_2, W_{Air} are weights opposing F_V

The line in orange is called the projection of the curved path.

Now Remember : F_H, F_V passes through Center of pressure

Example:

The dam below is a quarter circle 50m wide into the paper Determine F_H, F_V.



$\Sigma F_x = 0$

$F_1 = F_H = (\gamma_w)(A_{of\ projection})(h)$

specific weight of water — Area of projection — height of cg on projection

so $F_H = 9.807 \times 10^3 \times 20 \times 50 \times \frac{20}{2} = 98\ MN$

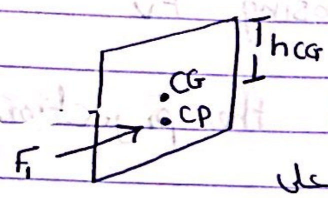
To find F_V:

$$F_V = W = mg = \rho V g$$
$$= \gamma V \rightarrow \text{volume of fluid}$$
(γ : specific weight)

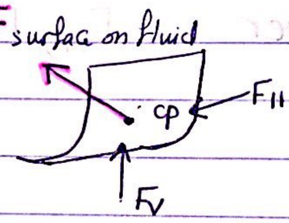
$$\text{so } F_V = (9.807\times10^3)(A)(50) = 9.807\times10^3 \times \left[\frac{1}{4}(\pi)(20)^2\right]\times 50$$
(A of parabola)
$$= 153.9\ MN$$

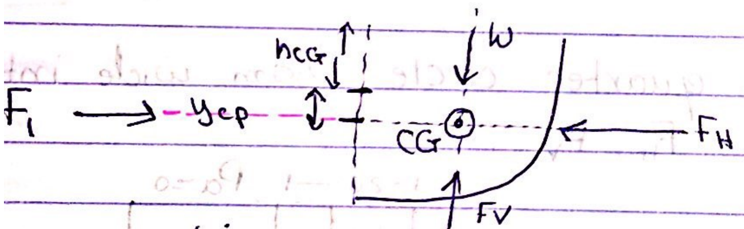
- Notes on solution:

ليش حطينا h_{CG} ضرب المسافة العمودية ؟
لأنه أنا باحسب القوة اللي على الـ Projection اللي تعاكس
F_H وهاي القوة بتأثر عمود السطح
بهذا الشكل



وبنفس الطريقة F_H بتمر في CP بس على الـ curved surface





فلو بدي ارسمها 2D

أما F_V فبتمر بـ CG الـ parapola لأنها بنفس
الخط اللي بتقع عليه W

2.7 Hydrostatic forces in layered fluids

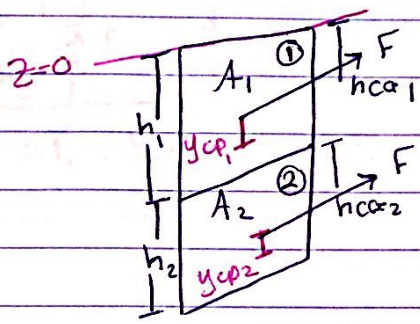
• بما أنه عندي طبقات مختلفة من موائع بـ ارتفاعات وكثافات مختلفة لا أستطيع استخدام 2.5

• نتعامل مع كل layer على حدا وبحل كل layer لوحدها

How to find F resultant ?

1 → you need to find P on each Area

First layer :- $P_1 = \gamma h_{ca_1}$



Second layer : $P_2 = \gamma h_1 + \gamma h_{ca_2}$

2 – $F = P A$

$F_1 = P_1 A_2$ $\qquad$ $F_2 = P_2 A_2$

$F = \Sigma F = F_1 + F_2$

3 → you need cp for each area

$$y_{cp_1} = \frac{\gamma \sin\theta \, I_{xx_1}}{F_1}$$

$$y_{cp_2} = \frac{\gamma \sin\theta \, I_{xx_2}}{F_2}$$

4 – Specify a reference $\boxed{z = 0}$

5 – find z_{cp} for each Area

$z_{cp_1} = h_{ca_1} + y_{cp_1}$

$z_{cp_2} = h_1 + h_{ca_2} + y_{cp_2}$

6- We find z_{cp} for the resultant force using Equation:

$$\sum F_i \, z_{cp_i} = F_R \, z_{cp_R}$$

$$F_1 \, z_{cp_1} + F_2 \, z_{cp_2} = (F_1 + F_2) \, z_{cp_R}$$ ← Find this

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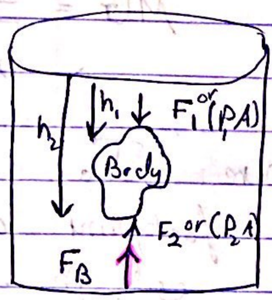
2.8 Bouyancy and stability

Bouyancy Force :-

$F_B = \boxed{\gamma\, v_{Body}}$ ← weight of fluid displaced by the object and F_B passes through center of Bouancy

F_B is the difference Between F_1 and F_2



- Where F_1, F_2 are weights difference of Body due to water weight and F_B is always upward fighting against gravity

$P_2 > P_1$ since $h_2 > h_1$

Example 2.11
Weight in air = 400N
" " water = 240N

نلاحظ انخفاض وزن الجسم في الماء

400 − 240 هذا الفرق هو وزن الماء المزاح و هو
= 160

وهذا الوزن يساوي F_B مقداراً ويعاكسه اتجاهاً

$$F_B = W = \gamma V$$
$$160 = 9.8\times10^3\, V$$
$$V = 0.016\, m^3$$

So: $\gamma_{Block} = \rho g = \frac{m}{V} g$, $m g = \frac{400}{g} g$; $= \frac{400}{0.016} = 24.5$ KN/m

Stability related to waterline Area :-

When you have a floating body tilting with Theta (θ) you can check the stability of this Body using equation

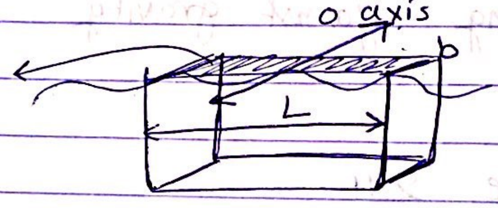
$$\overline{MG} = \frac{I_o}{V_{sub}} - \overline{GB}$$

: MG : metacentric height

Where, I_o : Area moment of inertia of the water line footprint around O axis

يعني الـ moment للمساحة اللي بتتحرك على سطح المية مثلاً

$$I_{o\,here} = \frac{bL^3}{12}$$



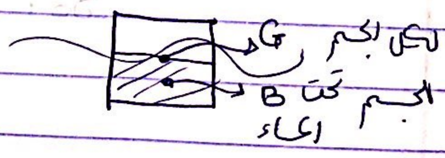
$V_{submerged}$ = مساحة الجسم تحت الماء

$\overline{GB}$: Distance Between Center of Gravity G and center of bouyancy B (B is center of Area under water)

Now : If $\overline{MG} > 0 \rightarrow$ Stable Body

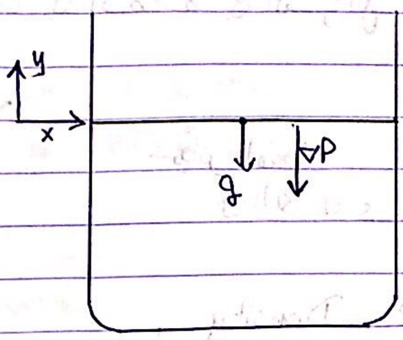
$\overline{MG} < 0 \rightarrow$ unstable Body

Note: B is found the same way & G is found But for Area (Body) under Water.



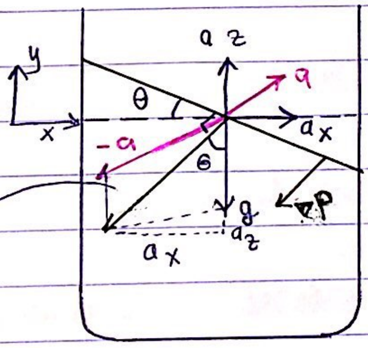
2.9: Uniform linear acceleration

Fluid at rest:

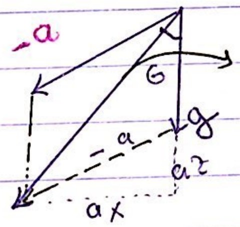


لما يكون للمائع في حالة الراحة تكون عليه تسارع الجاذبية فقط
وبالتالي تكون الزيادة في الضغط لأسفل اتجاهها ($\nabla P \propto g$)

Fluids moving a rigid bodies with an acceleration:-



اذا رأينا هذا ال Parallegram
عن قرب



$(\vec{g} - \vec{a})$

$$\tan\theta = \frac{a_x}{a_z + g}$$

لما يكون المائع تسارع كجسم يكون في تسارع اضافي وهو a هنا
التسارع هو محصلة لتسارعين: افقي a_x و عمودي a_z (مش دائماً موجود)
هذا التسارع اضافة إلى تسارع الجاذبية ليؤثران على زيادة الضغط
و ∇P تصبح عمودية على السطح المائل، باتجاه ال $\vec{g} - \vec{a}$: $\nabla P \propto (\vec{g} - \vec{a})$
يعني تصير $\nabla P = \rho(\vec{g} - \vec{a})$

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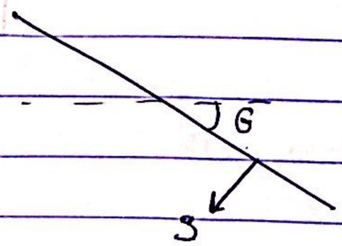
$$\frac{dP}{ds} = \rho G$$

هذه المعادلة تعني ان الزيادة
في الضغط في الاتجاه المائل $(g-a)$
تكون أكبر من الزيادة في الضغط في السوائل الساكنة

$\frac{dP}{ds}$ تغير الضغط في الاتجاه s



ρ: Density

$$\Rightarrow G = \frac{\text{محصلة}}{\text{التسارعات}} = \sqrt{a_x^2 + (g+a_z)^2}$$

$$P = \rho G \Delta s$$