Chapter 3:-

Integral Relations for a control volume:-

Systems vs control volumes

In a system:
- $m = \text{constant}$ $\quad \frac{dm}{dt} = 0$
- applied forces from the surroundings causes acceleration : $F = ma$
  (2nd law is applicable)
- Moments causes a rotation effect
  $M = \frac{dH}{dt}$, $H = \Sigma (r \times V) \delta m$

  where H: is the angular momentum.

- System's energy changes due to heat transfer and work

* In a control volume: None of these are applied instead we will use: The Raynolds Transport Theorem

- Mass is not constant and there are mass flow to and/or from

B : represents a particular property (extensive)
β : represents $\frac{B}{m}$: (intensive)

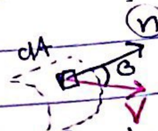
→ There are types of control volume:-
1. Fixed control volume
2. moving control volume → constant velocity / → variable velocity
3. deforming control volume

* n is a unit vector $|n| = 1$ Normal to dA

1- **Fixed**:-

Note:

- V & n
- Taking $V\cos\theta$ only



Now there are three changes happening to β:-

- A change within the CV : $\frac{d}{dt}\left(\int_{c.v} \rho \beta \, dV\right)$
- Outflow of β from the CV : $\int_{cs} \rho \beta V \cos\theta \, dA_{out}$
- inflow of β 〃 〃 〃 : $\int_{cs} \beta \rho V \cos\theta \, dA_{in}$

CV: Control volume.
CS: Control surface.

- The general form of the change within the System:-

$$① \text{———} \boxed{\frac{dB_{sys}}{dt} = \frac{d}{dt}\left(\int_{CV} \beta \rho \, dV\right) + \int_{cs} \beta \rho V \cos\theta \, dA_{out} - \int_{cs} \beta \rho V \cos\theta \, dA_{in}}$$

$\longrightarrow V \cdot n = V\cos\theta \, n = V\cos\theta = V_n$ (component of in direction of n)

$\dot{m} = \rho (V \cdot n) A$

$$\text{So } \frac{dB}{dt}_{sys} = \frac{d}{dt}\left(\int_{c.v} \beta \rho \, dV\right) + \int_{c.s} \beta \, d\dot{m}_{out} - \int_{cs} \beta \, d\dot{m}_{in}$$

2- **moving control volume** :

- with constant velocity: eq ① stays the same
  $\longrightarrow$ and V of control volume is ignored
- 〃 variable velocity: eq ① 〃 〃
  $\longrightarrow$ But V becomes a more complicated function.

3- deforming control volume :-

* $V_r = V - V_s$ ← vel of C.V

relative Vel

Fluid Vel

* $\frac{d}{dt}$ should be done after $\int$

$$\frac{dB}{dt} = \frac{d}{dt}\int_{C.V} \beta \rho \, dV + \int_{C.S} \beta \rho (V_r \cdot n) \, dA_{out} - \int_{C.S} \beta \rho (V_r \cdot n) \, dA_{in}$$

→ One dimensional flux :- flow properties are nearly uniform : $\int$ Becomes a sum (No need for integration)

→ Steady flow: change within CV = 0 $\left(\frac{d}{dt}\int_{C.V} \beta \rho \, dV = 0 \right)$

- In this chapter :- we will be finding a C.V form for :

1- Conservation of mass
2- linear momentum relation
3- Angular " "
4- energy equation

Conservation of mass:-

B = mass $\quad \beta = 1$

So

$$\left(\frac{dm}{dt}\right)_{sys} = \left(\frac{d}{dt}\int_{C.V} \rho \, dV\right) + \int_{C.S} \rho V_{r.n} \, dA_{out} - \int_{C.S} \rho V_{r.n} \, dA_{in}$$

$$\left(\frac{dm}{dt}\right)_{sys} = 0$$

and $\quad \frac{d}{dt}\int \rho \, dV = \int \frac{\partial \rho}{\partial t} dV$

→ and if the c.V has one.D out and inlets
Then

$$\boxed{0 = \int \frac{\partial \rho}{\partial t} dV + \sum(\rho V A)_{out} - \sum(\rho V A)_{in}}$$

- If the flow is steady, one Dimentional flux

$$\sum(\rho V A)_{out} = \sum(\rho V A)_{in}$$
$$\sum \dot{m}_{out} = \sum \dot{m}_{in}$$

- If the fluid is incompressible (ρ = constant)

$$\sum(VA)_{out} = \sum(VA)_{in}$$
$$\sum Q_{out} = \sum Q_{in} \quad (Q: \text{Volume flow} \ldots)$$

We can find V_{av}, ρ_{av} using

$$V_{AV} = \frac{1}{A}\int V.n \, dA \qquad \rho_{AV} = \frac{1}{A}\int \rho \, dA$$

$$(\rho V)_{AV} = \frac{1}{A}\int \rho (V.n) \, dA = \rho_{av} V_{av}$$

The linear momentum equation:

F: represents : surface Forces / Weights of mar

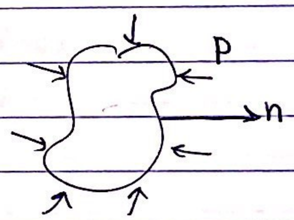
$B = mV \qquad \beta = V$

$$\frac{d(mV)}{dt}_{sys} = \sum F = \frac{d}{dt}\int_{c.v} (V\rho \, dV) + \int_{c.s} V\rho(V_r . n) \, dA$$

→ we can find $\sum F_x$, $\sum F_y$, $\sum F_z$ since the equation is a vector equation

Net pressure force on a closed control surface:-
If the surface of C.V has P_a affecting (unifor Value) then:

$F_{(up)} = 0$
uniform pressure



- $F_{press} = \int_{cs} (P - P_a)(-n)\, dA$
$= \int_{cs} P_{gage} (-n)\, dA$

- P, n has opposite directions

Momentum flux correction factor:-

for example:- $\sum F = \dot{m}(\beta_2 V_2 - \beta_1 V_1)$

β :- is called correction factor and It's close to a unity
- Dimensionless
- $\beta \geq 1$

For a steady flow, one Dimentional flux:-

$\sum F = \dot{m} V_{out} - \dot{m} V_{in}$

Hydrostatic
Pressure Weight External ← مشروحين في آخر التلخيص

Frictionless flow: the bernouli equation:-

For a steady frictionless incompressible flow along a Streamline: Bernouli's equation (conservation of energy in a fluid) is:-

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = \text{constant}$$

Restrictions.

1- Steady flow :

2- Incompressible flow : ρ is constant

3- frictionless flow لازم ما يكون في خسارة للطاقة على شكل حرارة أو شغل

4- flow along a single stream line

• نعتبر النقطتين اللي باخذهم على نفس المسار

Angular Momemntum theorem:-

$$\sum M_o = \frac{\partial}{\partial t}\left[\int_{C.V}(r\times V)\rho\, dV\right] + \int_{CS}(r\times V)\rho(V.n)\,dA$$

for a steady flow, one dimentional flux:-

$$\sum M_o = (r\times V)\dot{m}_{out} - (r\times V)\dot{m}_{in}$$

from forces

The energy equation:-

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\left(\int_{C.V} e\rho\, dV\right) + \int_{CS} e\rho(V.n)\,dA$$

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\left[\int_{CV}\left(\hat{u} + \frac{V^2}{2} + gz\right)\rho\, dV\right] + \int_{CS}\left(\hat{h} + \frac{V^2}{2} + gz\right)\rho(V.n)\,dA$$

one-Dimensional energy flux terms:-

the integration Goes away

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\left[\int_{CV}\left(\hat{u} + \frac{V^2}{2} + gz\right)\rho\, dV\right] + \sum\left(\hat{h} + \frac{V^2}{2} + gz\right)_{out}\dot{m}_{out}$$

$$- \sum\left(\hat{h} + \frac{V^2}{2} + gz\right)_{in}\dot{m}_{in}$$

for a steady flow, one dimentional flux:-

$(\dot{W}_v \approx 0)$

$$\dot{Q} - \dot{W}_s = \sum e\,\dot{m}_{out} - \sum e\,\dot{m}_{in}$$

$$\sum e = gz + \frac{1}{2}V^2 + h$$

Potential

Kinetic

Enthalpy (Thermal energy)

Friction and shaft work in low speed flow:

$$\left(\frac{p}{\gamma} + \frac{V_1^2}{2g} + z\right)_{in} = \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right)_{out} + h_{friction} - h_{pump} + h_{turbine}$$

→ h_f = friction head loss
→ h_p = pump head input
→ h_t = turbine head extraction

Additional Note :-

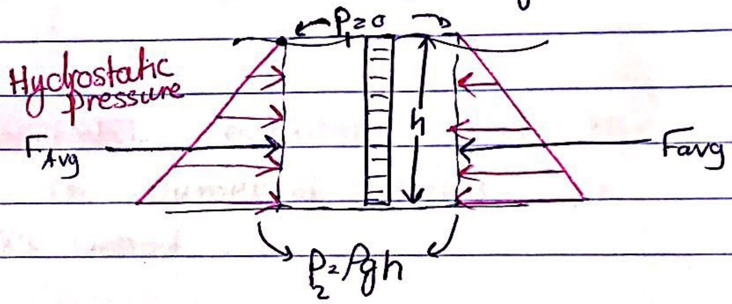
In the linear mementum equation : How to find $\sum F$

Forces can be :

1- External forces : Given in the Question
2- Weight : mg
3- Hydrostatic Force
4- Pressure Force

→ 3- Hydrostatic Force

- usually in gate Questions, the fluid will act on the walls of c.v causing hydro static pressure.



$$F_{avg} = \left(\frac{P_1 + P_2}{2}\right) A_{Avg}$$

$$= \frac{0 + \rho g h}{2} A$$

$$\boxed{F_{avg} = \frac{A \rho g h}{2}}$$

→ 4- Pressure Force

when a fluid is passing in a tube It acts on the walls of C.V causing pressure and the force resulting from this will equal:-

The pressure is alway action on A to inside (opposite direction of n)

$$F = (P_g A)\,(-n)$$

Direction only