

Chapter 5:-

Dimensional Analysis and Similarity

• **Dimensional analysis** is a **method** for reducing the number and complexity of experimental variables

• **Variables and constants:-**

→ **Dimensional variables** :- quantities that has dimensions and their numerical values may change. ex: speed, velocity, acc. ---

→ **Dimensional constants** :- quantities that has dimensions but its numerical values are constant. ex: g , Planck's constant ---

→ Pure constants, angles and revolutions are dimensionless

The Pi Theorem :-

The dimensionless groups found from the theorem are power products denoted by $\pi_1, \pi_2, \pi_3, \dots$

Steps of finding pi's :-

1- Find number of variables n

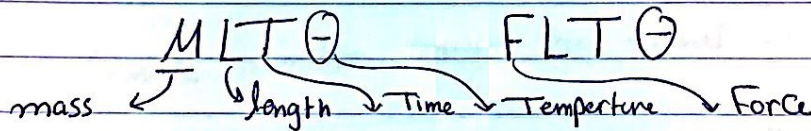
Ex: $P = f(F, A, \mu)$
 ① ② ③ ④

$n=4$ (P is included : Do NOT forget the first variable)

2- Find number of repeating variables j (P, F are Repeating)
 في كل مرة نستخدمها

3- Find number of pi's : $k = n - j = 4 - 2 = 2$
 A ← μ

5- Determine which system to use:



4- list the dimensions of each variable

حاصل
 القوت
 عشان تعرف
 كتار
 system

6- Take the first Group :-

Dimensionless

$$\Pi_1 = (A)^1 P^a F^b = FLT^0 \text{ or } MLT^0$$

$$\Pi_2 = (M)^1 P^a F^b = FLT^0 \text{ or } MLT^0$$

find a, b in each case

See Example 5.3

5.5. Modeling and Prototype

Model $\xrightarrow{\hspace{2cm}}$ Prototype

1. Geometrically similarity

2. Kinematic similarity

3. Dynamic similarity

1- The scale should be the same for all dimensions
Angles stay the same

2- Froude number must be equal

$$Fr_m = Fr_p$$

$$\frac{V_m^2}{g L_m} = \frac{V_p^2}{g L_p}$$

prototype \rightarrow $\frac{V_p^2}{g L_p} = Fr_p$
scale \rightarrow $\frac{V_m^2}{g L_m} = Fr_m$

$$L_m = \alpha L_p \quad \text{scale}$$

$$\sqrt{\alpha} = \frac{V_m}{V_p} = \left(\frac{L_m}{L_p} \right)^{1/2} = \frac{T_m}{T_p}$$

3- Dynamic Similarity :- Re and Mach equality

Compressible flow

Reynolds number and Mach number and specific heat ratio

Incompressible flow

No free surface

• Reynolds are equal

with a free surface

• Ray. equal
• Froude num
• Weber num
• Cavitation num

• Explanation of Ma , Re , Fr

① Mach number

$$Ma = \frac{V}{a}$$

Velocity

$$a = \sqrt{\frac{\gamma R T}{M}}$$

Temp
Gas. constant

$\frac{e p}{C_v}$

② Reynold's number:-

$$Re = \frac{\rho V L}{\mu}$$

Density

length / Diameter of inlet

Velocity

Viscosity

③ Froude number = $\frac{V}{\sqrt{2gL}}$

Combining Pi's Theorem and modeling

If the question is asking for a specific value of a property for a model or a prototype (with similarity: Dynamically and Geometrically)

- you can use Pi Theorem to find a law for the property and then use relation

$$\Pi_{\text{model}} = \Pi_{\text{prototype}} = \text{constant}$$