Conduction

- <u>Remember</u> :- $q''_x = \frac{q_x}{A} = -K \frac{dT}{dx}$ (1)

An alternative form of (1) is:-

$$\boxed{q''_x = \frac{q_x}{A} = -K \frac{\partial T}{\partial x}}$$

→ Partial diff.

→ T distribution Depends on x & t so we use ∂

The Heat Diffusion Equation

Recall energy conservation

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

For x :- $q_x + \dot{q}\, dx - q_{x+dx} = \rho\, c_p \frac{\partial T}{\partial t} dx$ (2)

Given → $q_x - q_{x+dx} = \frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right) dx$

So (2) Becomes:-

$$\frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right) \cancel{dx} + \dot{q}\, \cancel{dx} = \rho\, c_p \frac{\partial T}{\partial t} \cancel{dx}$$

$$\boxed{\frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right) + \dot{q} = \underbrace{\rho\, c_p}_{\text{can be replaced by } \frac{K}{\alpha}} \frac{\partial T}{\partial t}}$$

Special cases

[1] Steady-State conditions : $\rho c_p \frac{\partial T}{\partial t} = 0$

↳ So Diffusion equation becomes

$$\boxed{\dot{q} = -\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)}$$

[2] No energy generation : $\dot{q} = 0$

↳ So Diffusion equation becomes

$$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho c_p \frac{\partial T}{\partial t}}$$

If we are using other coordinates :-

Cylindrical :-
3- Directions

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Spherical:
3- Directions

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \phi}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)$$

$$+ \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Boundry Conditions

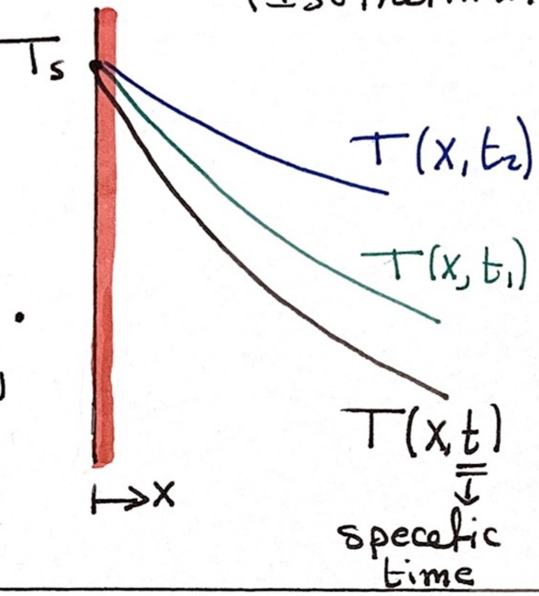
at $x=0$

1. Constant surface tempreture (Isothermal)

- at $x=0$, at any time

const. $\rightsquigarrow T_s = T(x=0, t)$

• معناها انو عند سطح $x=0$
الحرارة تساوي T_s في
أي زمن



2.a: Constant surface heat flux

→ Finite heat flux

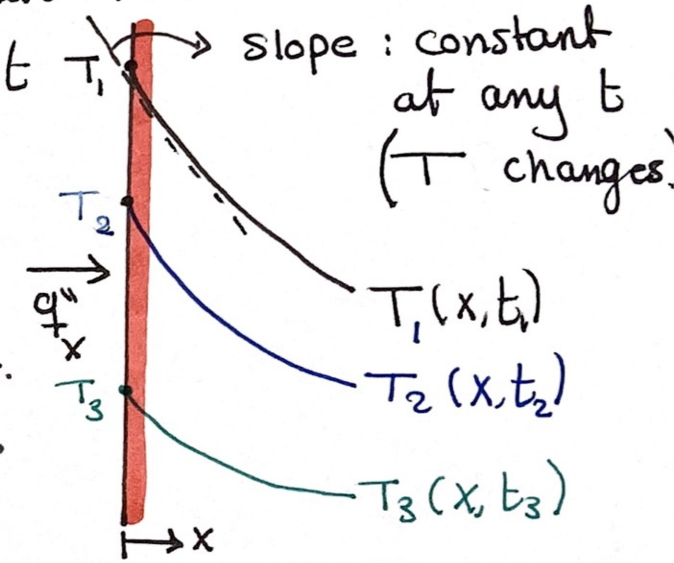
- at $x=0$, at any t

q''_x is constant

$$q''_x = -k \left.\boxed{\frac{\partial T}{\partial x}}\right|_{x=0}$$

Slope of tangent : const.

• معناها أنو التغير لـ T مع x
ثابت عند $x=0$ بالتالي q''_x ثابت

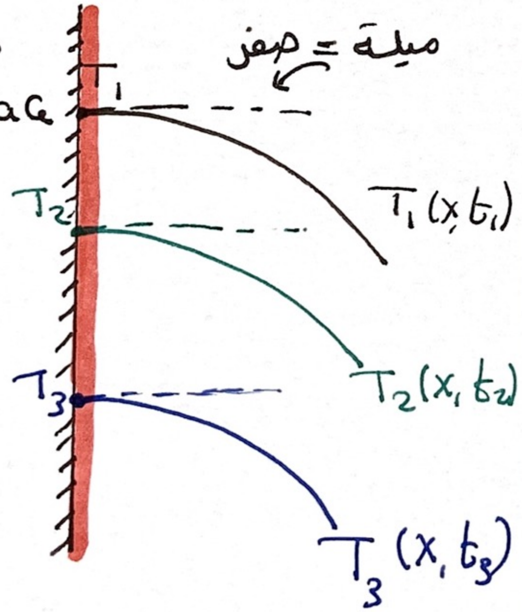


2.b: Constant Surface heat flux

→ Adiabatic or insulated surface

$$q''_x = 0 = \left.\frac{\partial T}{\partial x}\right|_{x=0}$$

• معناها أنو عند أي زمن او
درجة حرارة ال flux ثابت
وبساوي صفر



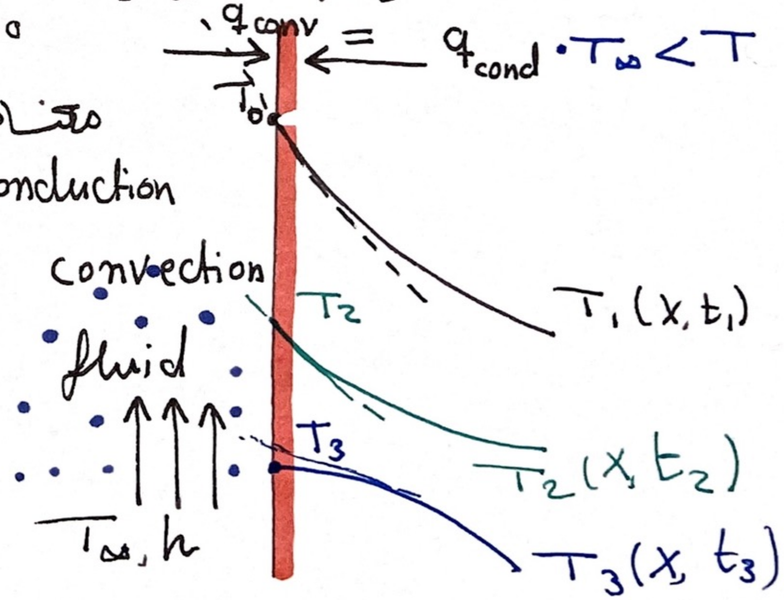
3. Convection surface condition

$$-k\left.\left(\frac{\partial T}{\partial x}\right)\right|_{x=0} = h\left[T_\infty - T(0, t)\right]$$

$q_{conv} = q_{cond}$

Note:
- $T_\infty > T_o$
- $T_\infty < T$

معناها أنو عند ال
conduction داخل الجدار
بساوي ال convection
من ال fluid



Thermal conductivity: (K)

$K_{solids} \gg K_{liquid} \gg K_{gas}$

- In Solids :-

$K = K_e + K_L$

free electron — lattice vibration

→ For Pure metals:-

$K_e = L_0 T / \rho_e$ → electrical resistivity

L_0 ↓ Lorenz number

→ non metallic solids:-

$K \rightarrow$ due to K_L

Thermal diffusity α

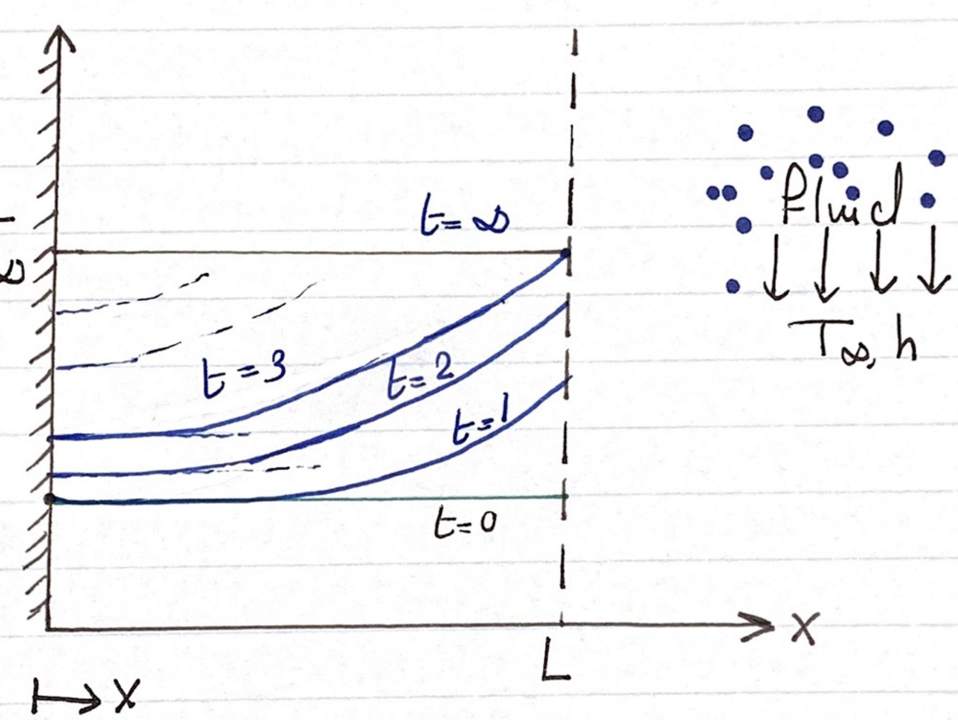
$$\alpha = \frac{k}{\rho c_p}$$

$\alpha_{metals} > \alpha_{non\ metals}$

Explaining $T(x,t)$, x curves

$T(x,t)$: Distribution of temprature

- Usually it is required to draw $T(x,t)$ vs x curve for conditions :- initial : $t \le 0$
  Steady-state :- $t \to \infty$
  two intermediate times
- Take Q 2.57 as an example



T_∞ ← بعد وقت طويل صارت درجة حرارة الحيط نفس درجة حرارة المائع

T_i ← يعني لما كان $t=0$ كانت الحرارة T_i في كل اجزاء الحيط

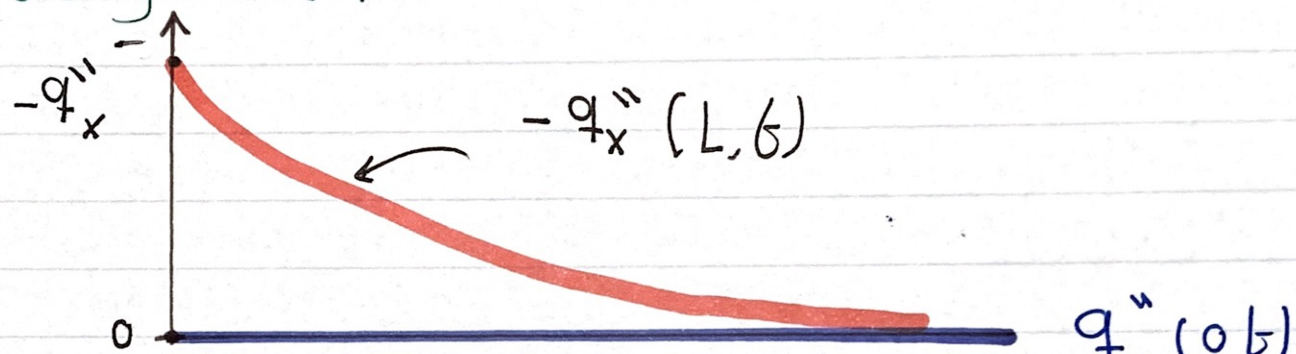
- Initially, $T = T_i$
- Suddenly, $x = L$ is heated
- lastly, It reached to a state in which $T_L = T_\infty$

Notice : at $x=0$ Insulation → الميل عند الـ $x=0$ دائماً صفر

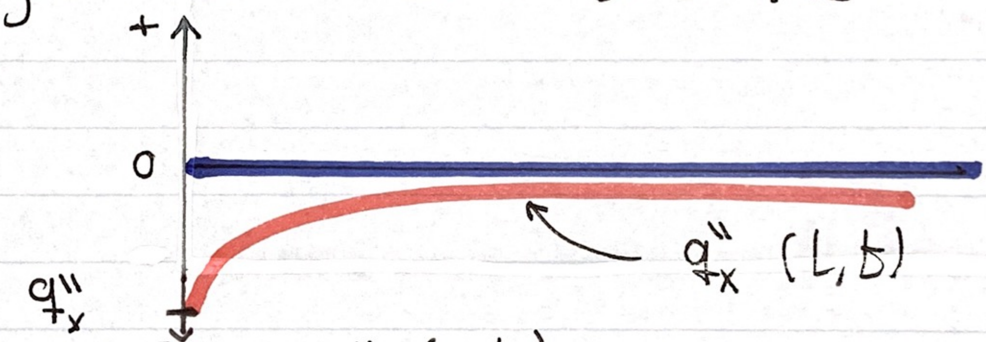
Explaining $q''_x(x,t)$ vs t curves

q''_x : flux distribution

- Usually it is required to draw $q''_x(x,t)$ vs t curve for conditions :- $x=0$, $x=L$
- Taking Q 2.57



or you can Draw it like this :-



* • First: taking $q''_x(0,t)$: at $x=0$, insulated so $q''=0$ with time T is const. because of the insulation

* • Second: taking $q_x(L,t)$, at $x=L$, It is heated so It gains heat, It starts at a max value (heat it gaind from fluid) and decreases until it reaches zero when $T_L = T_\infty$