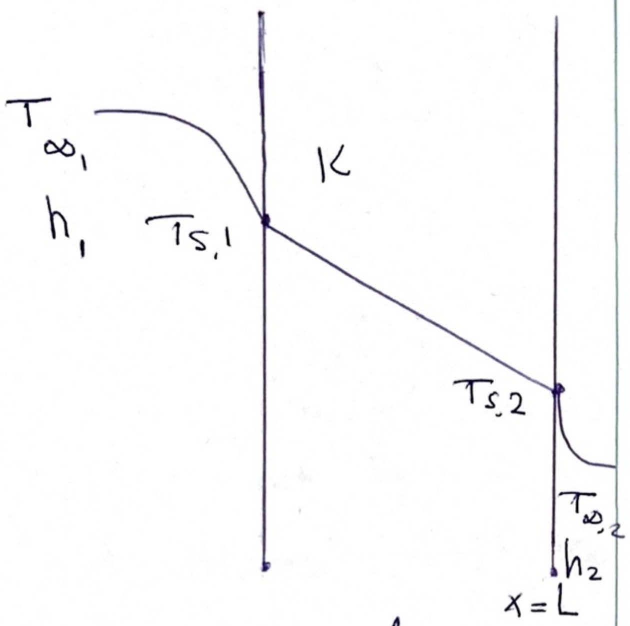
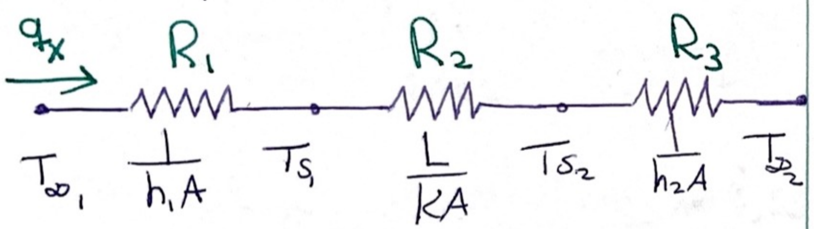
Plane Wall



- Equivalent thermal circuit



General Solution (No $\dot{q}$, S.S)

$$T(x) = (T_{s_2} - T_{s,1})\frac{x}{L} + T_{s,1}$$

(linear)

Fourier's law

$$\underbrace{q''_x = \frac{q_x}{A}}_{\text{constant at any } x} = \frac{K}{L}(T_{s_1} - T_{s_2})$$

Thermal Resistance

$$R_{t,cond} = \frac{T_{s_1} - T_{s_2}}{q_x} = \frac{L}{KA}$$

$$R_{t\,conv} = \frac{T_s - T_\infty}{q_x} = \frac{1}{hA}$$

Note:

q is constant (like current)

so,

$$q_x = \frac{T_{\infty,1} - T_{s_1}}{1/h_1 A} = \frac{T_{s_1} - T_{s_2}}{L/KA} = \frac{T_{s_2} - T_{\infty_2}}{1/h_2 A}$$

$$q_x = \frac{T_{\infty_1} - T_{\infty 2}}{R_{tot}} = \frac{T_{\infty_1} - T_{\infty 2}}{R_1 + R_2 + R_3}$$

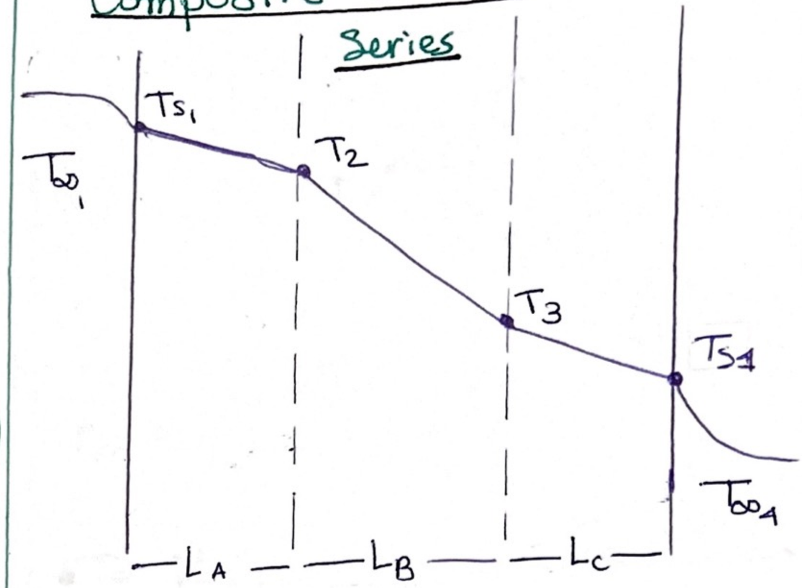
- If radiation is took into consideration then:

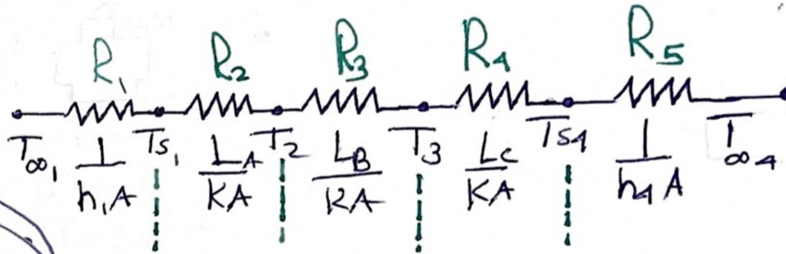
$$R_{t,rad} = \frac{T_s - T_{sur}}{q_{rad}} = \frac{1}{h_r A}$$

$$q''_x = \frac{T_{\infty_1} - T_{\infty 2}}{R_{tot}}$$ (But $R_{1,3}$ becomes $\frac{1}{h_1}$ and R_2 becomes $\frac{L}{K}$)

Composite wall

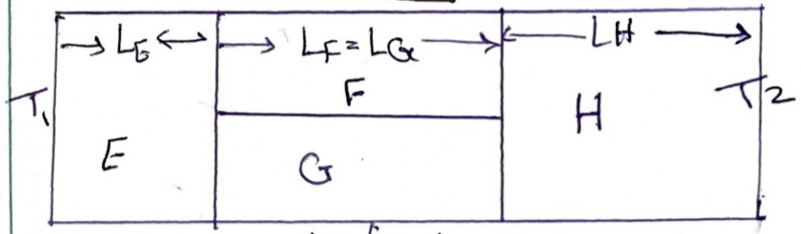
Series

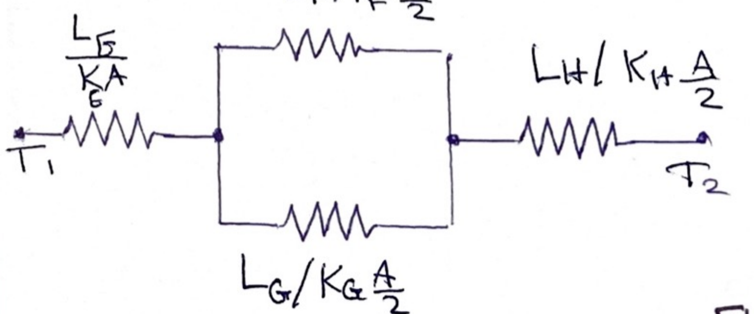




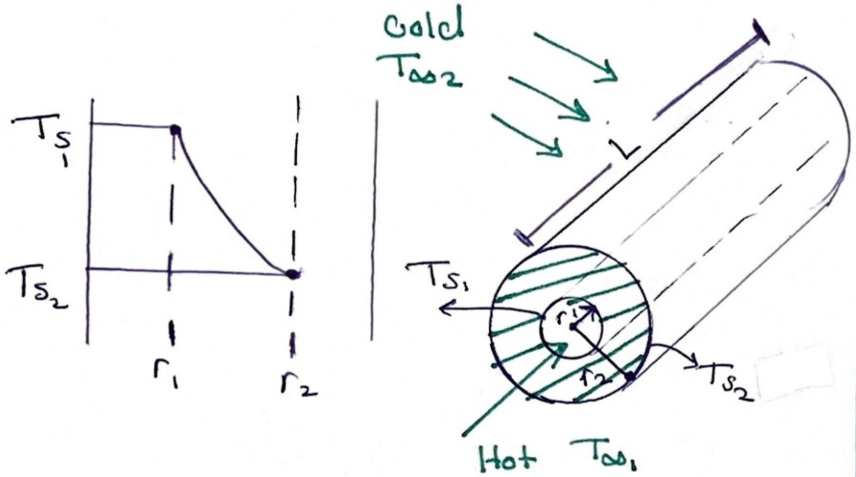
- Same rules are applied

Parallel

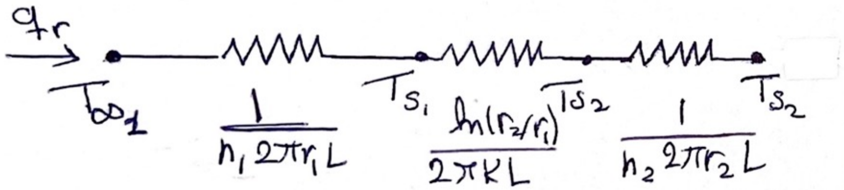




Cylinder



- Equivalent thermal Resistance



Temprature distribution No $\dot{q}$, SS

$$T(r) = \frac{T_{s_1} - T_{s_2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

fourier's law

$$q_r = \frac{2\pi L K (T_{s_1} - T_{s_2})}{\ln(r_2/r_1)}$$

constant at any r

Thermal Conductivity

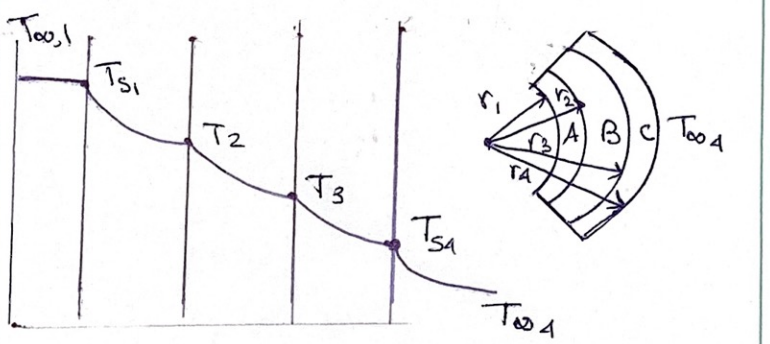
$$R_{t\,cond} = \frac{\ln(r_2/r_1)}{2\pi L K}$$

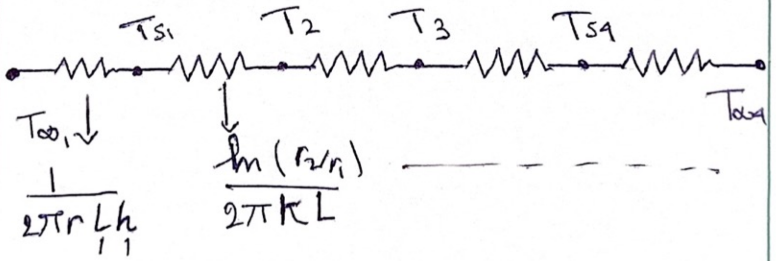
$$R_{t\,conv} = \frac{1}{2\pi r L h}$$

Note:
q is constant (like current)

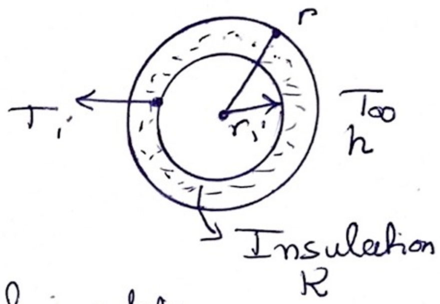
$$q_r = \frac{T_{\infty_1} - T_{\infty_2}}{R_{tot}}$$

Composite cylindrical wall





Critical insulation Radius



$$r_{cr} = \frac{K}{h}$$ ← of insulation / of fluid

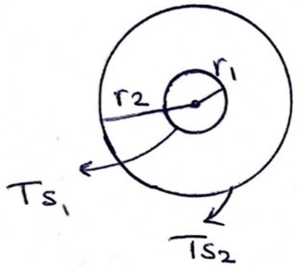
- To Know if an insulation can be added:-

$r_i < r_c = \frac{K}{h}$ → q increases with increasing r (insulation)

$\boxed{\text{Don't add Insulation}}$

If : $r_i > r_{cr}$ → q decreases with increasing r (insulation)

Sphere



fourier's law

$$q_r = \frac{4\pi K \,(T_{s_1} - T_{s_2})}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Thermal conductivity

$$R_{t,cond} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

critical Radius

$$r_{cr} = \frac{2K}{h}$$

Heat generation $\dot{q} \neq 0$

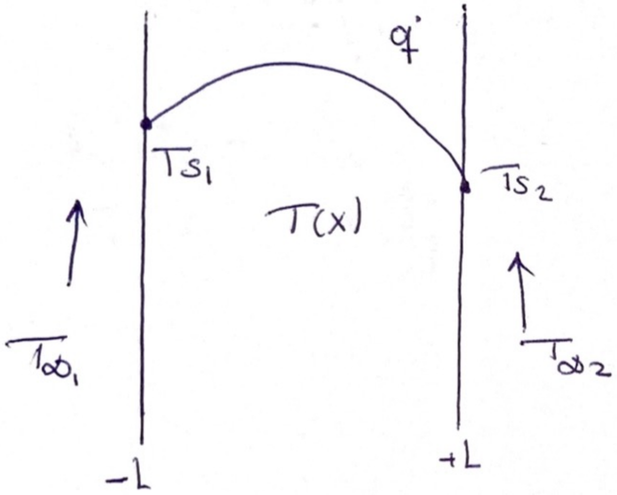
- conversion from electrical to thermal energy (common)

$$\dot{q} = \frac{\dot{E}_g}{V} = \frac{I^2 R_e}{V}$$

- Endothermic and Exothermic reactions
- Conversion from electromagnetic to thermal energy

Energy equation :-
$E_{in} - E_{out} = \dot{q}$

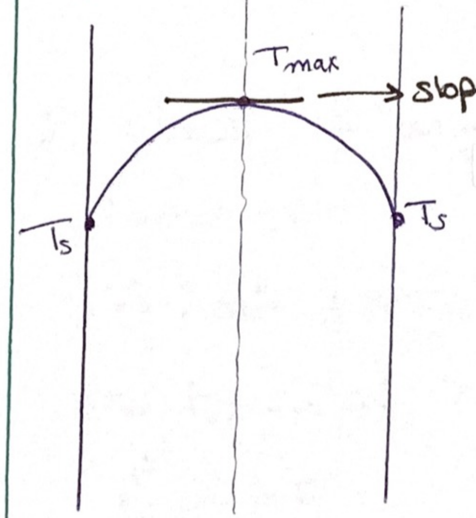
Plane wall



General solution

$$T(x) = \frac{\dot{q}L^2}{2K}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s_2} - T_{s_1}}{2}\frac{x}{L} + \frac{T_{s_1} + T_{s_2}}{2}$$

if $T_{s_1} = T_{s_2} = T_s$



$\longrightarrow$ slope $= 0 \rightarrow \frac{dT}{dx} = 0$

So considerd to be adiabatic

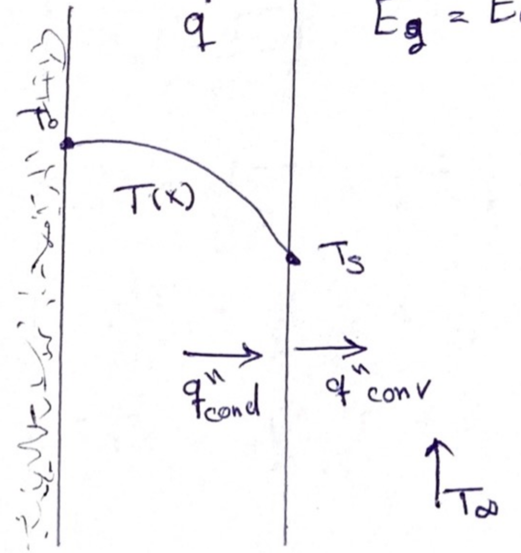
$\Longrightarrow$

Solution

$$T(x) = \frac{\dot{q}L^2}{2K}\left(1 - \frac{x^2}{L^2}\right) + T_s$$

$\dot{E}_{in} = 0 \rightarrow$ Energy equation

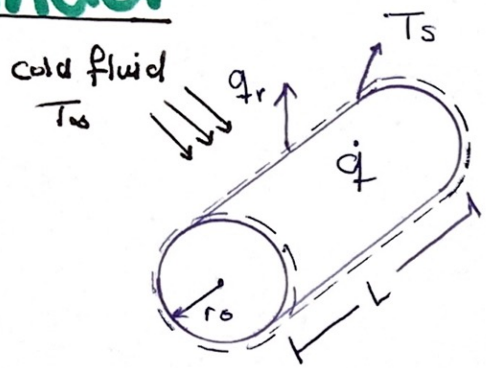
$\dot{E}_g = \dot{E}_{out}$



$q''_{cond} = q''_{conv}$

$\hookrightarrow T_s = T_\infty + \frac{\dot{q}L}{h}$

Cylinder



$$T(r) = \frac{\dot{q} r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$$

Relating T_∞ to T_s

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h}$$

How to derive an equation for any system :-

1. start at energy equation (general)

$$\dot{E}_{in} - \dot{E}_{out} + \underbrace{\dot{q}V}_{\dot{E}_g} = 0$$

2. check if any of these terms $= 0$

No heat generation : $\dot{q} = 0$

one insulated surface $\dot{E}_{in}$ or $\dot{E}_{out} = 0$

3. let's take radial system as an Example

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{dT}{dr}\right)} + \dot{q} = 0$$

4. Integrate equation after organizing it

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = \dot{q}$$

$$\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = r\dot{q}$$

$$d\left(kr\frac{dT}{dr}\right) = r\dot{q}\,dr$$

$$kr\frac{dT}{dr} = \frac{r^2}{2}\dot{q} + c_1$$

$$\frac{dT}{dr} = \frac{r\dot{q}}{2k} + \frac{c_1}{r}$$

$$T(r) = \frac{r^2\dot{q}}{4k} + \ln r\, c_1 + c_2$$

5. we choose two values of r to solve equation and if there is an insulation, we use

$$\left.\frac{dT}{dr}\right|_{r=r_i} = 0 \quad (r_i = \text{surface of insulation})$$