Chapter 8 Internal flow

For flowing fluid

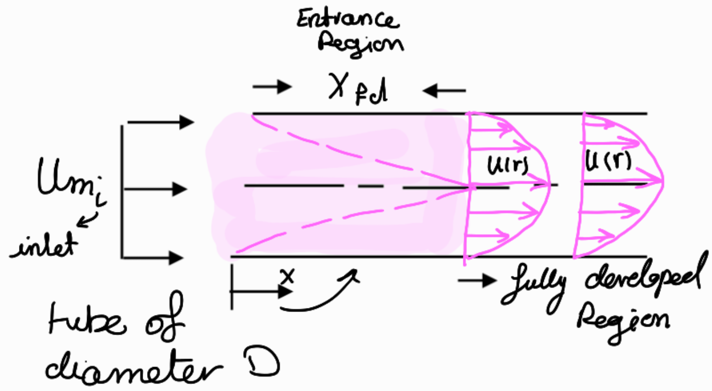
$$\boxed{Re_D = \frac{U_m D}{\nu}}$$

$\dot{m} = \rho V A$, Velocity $= U_m$

$\mu = \rho \nu$

For tube flow:

- If $Re_D < 2300$: laminar
- If $Re_D \geq 2300$: turbulent



X_{fd}: distance to fully developed Region

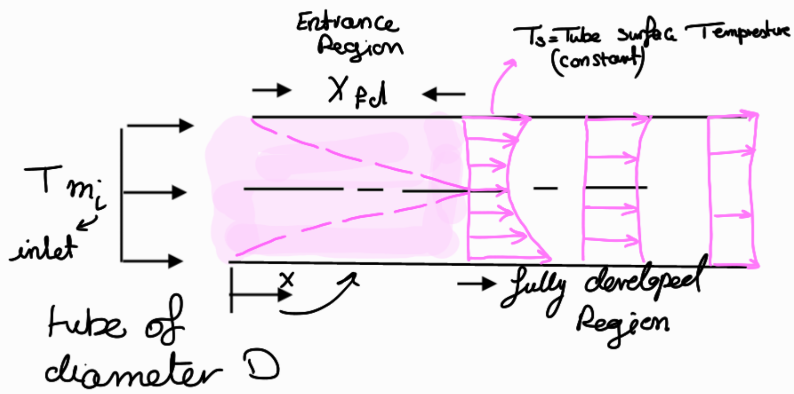
Note, $Re_D = \frac{4\dot{m}}{\pi D \mu}$

For temperature flow

$$\left(\frac{X_{fd}}{D}\right)_{lam} = 0.05\, Re_D\, Pr$$

$$\left(\frac{X_{fd}}{D}\right)_{Turb} = 10$$

Always calculate it

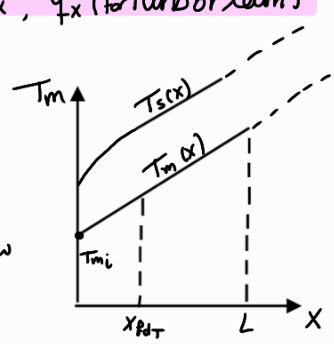


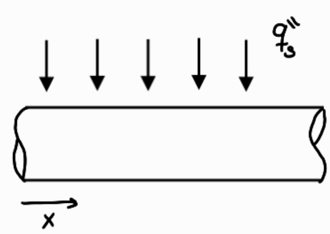
Constant surface Heat flux, q_x'' (for Turb or lam)

$$T_m(x) = T_{mi} + \frac{q_s'' P}{\dot{m} c_p} X$$

when $X=0$, $T_m = T_{mi}$

$T_{s,o} = T_{m,o} + \frac{q''}{h}$ → for the flow

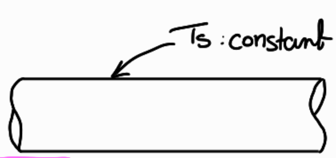


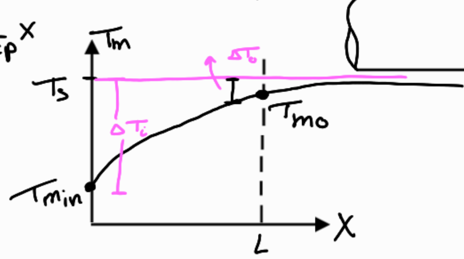


Constant surface temperature (for Turb or lam)

$$\frac{T_s - T_m(x)}{T_s - T_{mi}} = e^{-\frac{P\bar{h}}{\dot{m} c_p} X}$$

$P = \pi D$



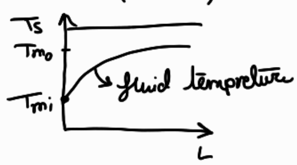


$\to$ log mean temprerature difference

$$q = \bar{h} A_s \Delta T_{ml} \qquad \text{where} \qquad \Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

Also

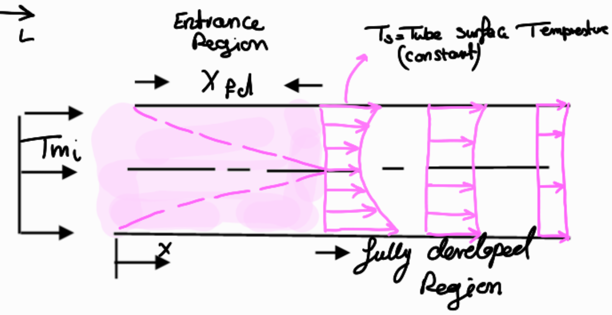
$$q = \dot{m}\, c_p \,(T_{mo} - T_{mi}) \to q_s \text{ const or } T_s \text{ const}$$



Arithmetic mean temprerature difference:

$$\Delta T_{am} = \frac{\Delta T_o + \Delta T_i}{2} = \frac{(T_s - T_o) + (T_s - T_i)}{2}$$

fluid



To find $\bar{h}$ for tubes (laminar flow)

- For q_s'' constant

At T_m $\to$ $Nu_D = \frac{hD}{k} = 4.34$ $\to$ This is for:
- fully developed conditions
- fully developed velocity
- fully developed temperature

- For T_s constant

At T_m $\to$ $Nu_D = \frac{hD}{k} = 3.66$

- For entrance Region (Thermal entry)

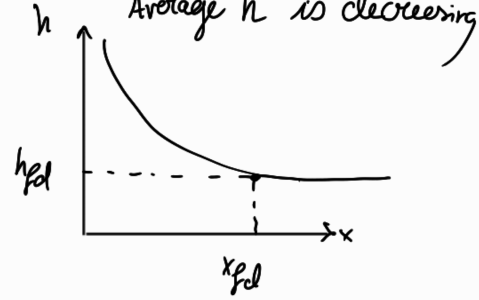
$$\bar{Nu}_D = \frac{\bar{h}D}{k} = 3.66 + \frac{0.0668\, Gz_D}{1 + 0.04 (Gz_D)^{2/3}} \qquad \text{where} \qquad Gz_D = \left(\frac{D}{x}\right) Re_D \, Pr$$

$\to$ Graetz number

T_s = constant

Properties at $\bar{T}_m = \frac{T_{mi} + T_{mo}}{2}$

Note



Combined entery length

$$\bar{Nu}_D = \frac{\dfrac{3.66}{\tanh\left[2.264\, Gz_D^{-1/3} + 1.7\, Gz_D^{-2/3}\right]} + 0.0499\, Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432\, Pr^{1/6} Gz_D^{-1/6})}$$

To find $\bar{h}$ for tubes (Turbulent flow) $\to$ smooth (fully developed)

$$\bar{Nu}_D = 0.023\, Re_D^{4/5}\, Pr^n$$

$n = 0.4$ for heating $T_s > T_m$

$n = 0.3$ for cooling $T_s < T_m$

- Prop at $\bar{T}_m$
- Used for T_s const or q_s constant

If the tube is rough

$$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$

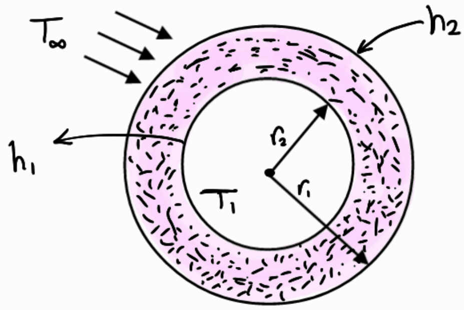
f : from moody chart

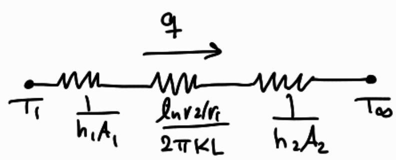
valid for $3000 \le Re_D \le 5\times10^6$

Overall heat transfer coefficient

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = e^{\left(\frac{-\bar{U} A_s}{\dot{m} c_p}\right)} \rightarrow A_s = \pi D L$$

$$q = \bar{U} A_s \Delta T_{lm}$$





Noncircular Tubes

hydraulic diameter

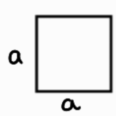
$$D_h = \frac{4A_c}{P}$$

$A_c \rightarrow$ flow cross-sectional area

$P \rightarrow$ wetted perimeter

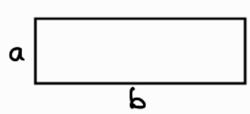
$D_h \rightarrow$ used instead of D

- For square section



$D_h = a$

- For rectangular section



$$D_h = \frac{2ab}{a+b}$$

- Parallel sheets $b \gg a$

$D_h = 2a$

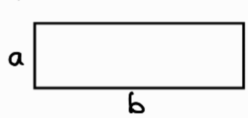
Table 8.1 Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

Cross Section	$\frac{b}{a}$	$Nu_D = \frac{hD_h}{k}$ (Uniform q_s'')	$Nu_D = \frac{hD_h}{k}$ (Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
a, b	1.0	3.61	2.98	57
a, b	1.43	3.73	3.08	59
a, b	2.0	4.12	3.39	62
a, b	3.0	4.79	3.96	69
a, b	4.0	5.33	4.44	73
a, b	8.0	6.49	5.60	82
Heated	∞	8.23	7.54	96
Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

Used with permission from W. M. Kays and M. E. Crawford, *Convection Heat and Mass Transfer*, 3rd ed. McGraw-Hill, New York, 1993.

$$Re = \frac{\dot{m} D_h}{A_c \mu} = \frac{\dot{m}\left(\frac{2ab}{a+b}\right)}{ab\,\mu}$$

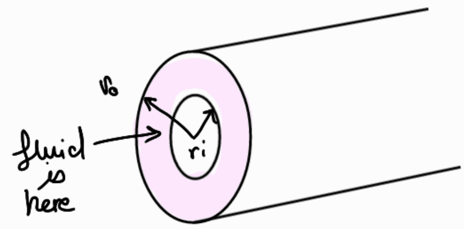
$A_c \rightarrow$ cross sectional area



For cases not in the table, use D_h in previous correlations obtained

But P, A are calculated for original shap

Concentric tube annulus



For interior:

$$q''_i = h_i\,(T_{si} - T_m)$$

Tables 8.2, 8.3

$$Nu_i = \frac{h_i D_h}{K}$$

For outside

$$q''_o = h_o\,(T_{so} - T_m)$$

$$Nu_o = \frac{h_o D_h}{K}$$

$$\boxed{D_h = D_o - D_i}$$ (hydraulic diameter)

Generally $q = \dot{m}\, c_p\,(T_{m,o} - T_{m,i})$

Table 8.2 Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o	Comments
0	—	3.66	See Equation 8.55
0.05	17.46	4.06	
0.10	11.56	4.11	
0.25	7.37	4.23	
0.50	5.74	4.43	
≈1.00	4.86	4.86	See Table 8.1, $b/a \to \infty$

Heated

Insulated

Note
$$q'' = \frac{q}{A} = \frac{q' L}{A}$$
$$A = 2\pi r L$$

Table 8.3 Influence coefficients for fully developed laminar flow in a circular tube annulus with uniform heat flux maintained at both surfaces

D_i/D_o	Nu_{ii}	Nu_{oo}	θ_i^*	θ_o^*
0	—	4.364[a]	∞	0
0.05	17.81	4.792	2.18	0.0294
0.10	11.91	4.834	1.383	0.0562
0.20	8.499	4.833	0.905	0.1041
0.40	6.583	4.979	0.603	0.1823
0.60	5.912	5.099	0.473	0.2455
0.80	5.58	5.24	0.401	0.299
1.00	5.385	5.385[b]	0.346	0.346

Used with permission from W. M. Kays and H. C. Perkins, in W. M. Rohsenow and J. P. Hartnett, Eds., *Handbook of Heat Transfer*, Chap. 7, McGraw-Hill, New York, 1972.

- If the tube is placed in a fluid

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = e^{-\left(\frac{\bar{U} A_s}{\dot{m}\, c_p}\right)}$$

$$q = \bar{U} A_s \,\underbrace{\Delta T_{lm}}_{\text{log mean}}$$