Chapter 9 Free Convention

Expansion coefficient

$$\beta \simeq -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T}$$ obtained from Table A.5, A.6

- For an ideal gas

$$\beta = \frac{1}{T_f} \qquad T_f = \frac{T_s + T_\infty}{2}$$

Grashof number

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

Rayleigh number

$$Ra_{x,c} = Gr_{x,c}\,Pr = \frac{g\beta(T_s - T_\infty)X^3}{\nu\alpha}$$

if no x is specified:
use $X = L$ الارتفاع length

$$Ra_{x,c}\begin{cases} \le 10^9 & \text{laminar} \\ > 10^9 & \text{Turbulent}\end{cases}$$

→ For vertical Plate

> **Note:**
> All properties evaluated at T_f : film temp
> $$T_f = \frac{T_s + T_\infty}{2}$$

Empirical correlations

- if laminar or Turbulent

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = C\,Ra_L^n \quad : n\begin{cases} 1/4 & \text{laminar} \\ 1/3 & \text{Turbulent}\end{cases}, \; C\begin{cases} 0.59 & \text{laminar} \\ 0.1 & \text{turbulent}\end{cases}$$

$$Ra_L = Gr_L\,Pr = \frac{\overset{9.81}{g}\,\overset{\text{Tables}}{\beta}\,(T_s - T_\infty)L^3}{\nu\alpha}$$

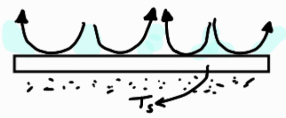
To find q, we calculate radiation: only for air

$$q = q_{conv} + q_{rad} \quad : \quad q_{rad} = \varepsilon \underbrace{A_s}_{\text{plate surface area}} \sigma (T_s^4 - T_{sur}^4)$$

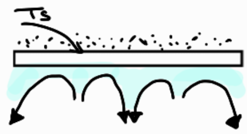
→ **For Horizontal plate** rectangulars, circles and squares

Horizontal plate

Upper Surface of Hot plate
or lower surface of cold plate



Hot plate, $T_s > T_\infty$



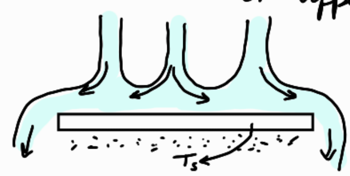
Cold plate, $T_s < T_\infty$

$\overline{Nu}_L = 0.54\, Ra_L^{1/4} \longrightarrow 10^4 \le Ra_L \le 10^7, \quad Pr \ge 0.7$

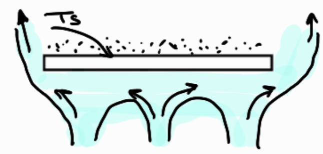
$\overline{Nu}_L = 0.15\, Ra_L^{1/3} \longrightarrow 10^7 \le Ra_L \le 10^{11}, \quad \text{all } Pr$

Direction of heat transfer is upward

lower Surface of Hot plate
or Upper surface of cold plate



Cold plate, $T_s > T_\infty$



Hot plate, $T_s < T_\infty$

$\overline{Nu}_L = 0.52\, Ra_L^{1/5} \longrightarrow 10^4 \le Ra_L \le 10^9, \quad Pr \ge 0.7$

Direction of heat transfer is downward

$L = \frac{A_s}{P}$ — A_s → surface area of one side; P → perimeter

- For a circular disk $L = \frac{R}{2}$
- For a square Surface $L = \frac{a}{4}$
- For a rectangular Surface $L = \frac{\text{length} \times \text{width}}{2(\text{length} + \text{width})}$
- For a rectangular Surface with width much smaller th

Long horizontal Cylinder

$\overline{Nu}_D = \frac{\bar{h} D}{k} = C\, Ra_D^n$ c,n from Table 9.1

Table 9.1 Constants of Equation 9.33 for free convection on a horizontal circular cylinder [22]

Ra_D	C	n
10^{-10}–10^{-2}	0.675	0.058
10^{-2}–10^{2}	1.02	0.148
10^{2}–10^{4}	0.850	0.188
10^{4}–10^{7}	0.480	0.250
10^{7}–10^{12}	0.125	0.333

Sphere

$$\overline{Nu}_D = 2 + \frac{0.589\, Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}}$$

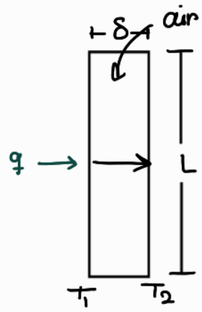
$L = D$ for sphere

$A_s = 4\pi r^2$

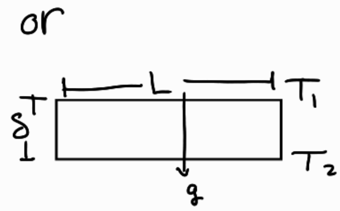
Enclosure

$$\frac{\bar{h}\delta}{k} = C(\underbrace{Gr_\delta Pr}_{Ra^n})^n \left(\frac{L}{\delta}\right)^m$$

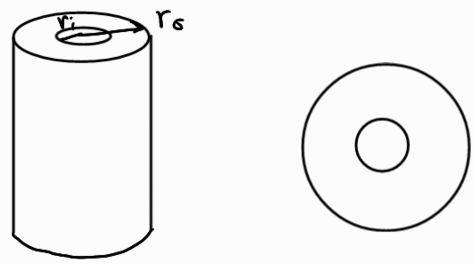
where: $Gr_\delta = \dfrac{g\beta(T_1-T_2)\delta^3}{\nu^2}$



or



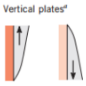
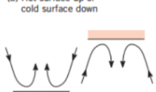
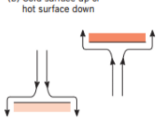
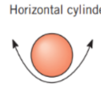
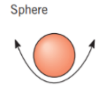
- If there are two cylinders
  Same but $\delta = r_o - r_i$



- If there are two spheres
  Same but $\delta = r_o - r_i$

Fluid	*Geometry*	$Gr_\delta Pr$	Pr	$\frac{L}{\delta}$	*C*	*n*	*m*
Gas	Vertical plate,	<2000	$k_e/k = 1.0$				
	isothermal	6000–200,000	0.5–2	11–42	0.197	$\frac{1}{4}$	$-\frac{1}{9}$
		200,000–1.1×10^7	0.5–2	11–42	0.073	$\frac{1}{3}$	$-\frac{1}{9}$
	Horizontal plate,	<1700	$k_e/k = 1.0$				
	isothermal heated from	1700–7000	0.5–2	—	0.059	0.4	0
	below	7000–3.2×10^5	0.5–2	—	0.212	$\frac{1}{4}$	0
		$>3.2 \times 10^5$	0.5–2	—	0.061	$\frac{1}{3}$	0
Liquid	Vertical plate,	<2000	$k_e/k = 1.0$				
	constant heat	10^4–10^7	1–20,000	10–40	Eq. 7-52	—	—
	flux or	10^6–10^9	1–20	1–40	0.046	$\frac{1}{3}$	0
	isothermal						
	Horizontal plate,	<1700	$k_e/k = 1.0$	—			
	isothermal,	1700–6000	1–5000	—	0.012	0.6	0
	heated from	6000–37,000	1–5000	—	0.375	0.2	0
	below	37,000–10^8	1–20	—	0.13	0.3	0
		$>10^8$	1–20		0.057	$\frac{1}{3}$	0
Gas or liquid	Vertical annulus	Same as vertical plates					
	Horizontal annulus,	6000–10^6	1–5000	—	0.11	0.29	0
	isothermal	10^6–10^8	1–5000	—	0.40	0.20	0
	Spherical annulus	120–1.1×10^9	0.7–4000		0.228	0.226	0

TABLE 9.2 Summary of free convection empirical correlations for immersed geometries

Geometry	Recommended Correlation	Restrictions
1. Vertical plates[a] 	Equation 9.26	None
2. Inclined plates Cold surface up or hot surface down 	Equation 9.26 $g \to g\cos\theta$	$0 \le \theta \lesssim 60°$
3. Horizontal plates (a) Hot surface up or cold surface down 	Equation 9.30 Equation 9.31	$10^4 \lesssim Ra_L \lesssim 10^7$, $Pr \gtrsim 0.7$ $10^7 \lesssim Ra_L \lesssim 10^{11}$
(b) Cold surface up or hot surface down 	Equation 9.32	$10^4 \lesssim Ra_L \lesssim 10^9$, $Pr \gtrsim 0.7$
4. Horizontal cylinder 	Equation 9.34	$Ra_D \lesssim 10^{12}$
5. Sphere 	Equation 9.35	$Ra_D \lesssim 10^{11}$ $Pr \gtrsim 0.7$

[a] The correlation may be applied to a vertical cylinder if $(D/L) \gtrsim (35/Gr_L^{1/4})$.

Note:

In Blue these are treated like horizontal plates with rectangular crosssection