

Exp 9: Impedance and Sinusoidal Steady State

* For a Sinusoidal Source:-

$$V = V_m \cos(\omega t + \phi)$$

$$I = I_m \cos(\omega t + \theta)$$

• V_m, I_m is Max Amp

$$V = V_m e^{i\theta} = \text{Phasor} \left[\text{form} \right] \left\{ V_m \cos(\omega t + \theta) \right\}$$

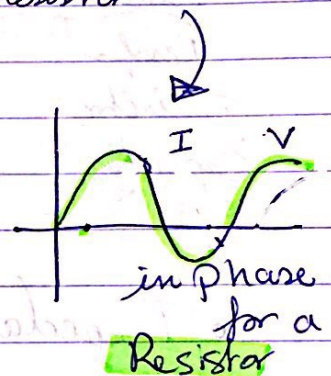
• Ohm's law for a Resistor $V = R I_m e^{i\theta}$ But $\theta = 0$ for a Resistor

• V-I Relation for an Inductor

$$V = L \frac{di}{dt}$$

$$\text{So } V = L \frac{di}{dt} = L \frac{d(I_m \cos(\omega t + \theta_i))}{dt}$$

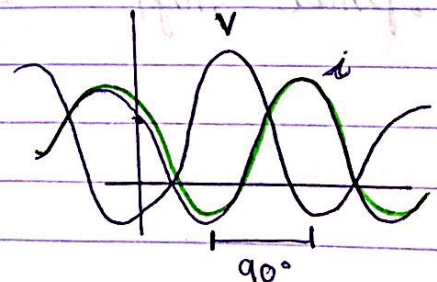
$$= -\omega L I_m \sin(\omega t + \theta_i) = -\omega L I_m j$$



• Voltage and Current are out of phase by 90°

$$\text{So } V = \omega L I_m \angle (90 + \theta_i)$$

Current lags voltage



* V-I Relation for a Capacitor :-

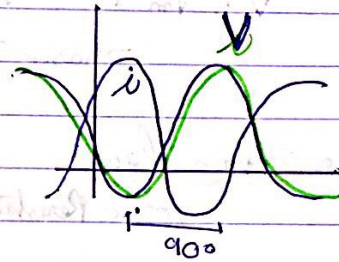
$V = V_m \cos(\omega t + \phi_v)$ and $i = C \frac{dv}{dt}$

So $V = \frac{1}{j\omega C} \cdot I$ (Current lags Voltage by 90°)

$V = I_m / \omega C \angle (-90^\circ)$

• Impedance and Reactance :-

$V = Z I$



Element	Impedance	Reactance
Resistor	R	-
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

• Impedance is Not a phasor but It's a complex Number

• It is the Quantity analogous to resistance, capacitance and inductance in the time domain

• Imaginary part of $Z =$ Reactance

• phase shift: $\Delta\phi = 360^\circ \times f \times \Delta t$

