Degrees of freedom :-

Def: number of Independant parameters needed to fully control the joint.

- Rigid body in a plane DoF = 3
- Rigid body in space DoF = 6

Types of Joints :

1- Revolute .J : Ex : Hinge , Pin DoF = 1

2- Prismatic .J :

or

DoF = 1

3- Cylindrical J:

DoF = 2

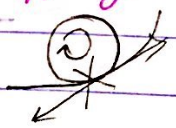
4- Spherical .J:

DoF = 3

5- Helical . J:

DoF = 1

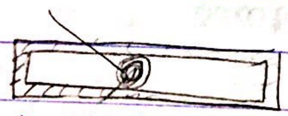
6- Contant . J: • Rolling → without slipping DoF = 1



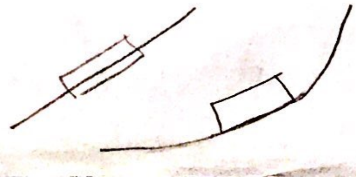
Contant in a point : DoF = 1
" " a line : DoF = 2

→ with slipping DoF = 2

Ex : pin iside a slot



• slipping → without Rolling DoF = 1



Mobility of a mechanism : (m)

- J_1 : one Degree of freedom
- J_2 : Two Degrees of freedom

$m = 3(n-1) - 2J_1 - J_2$ (In plane).

$m = 6(n-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4$ (In space)

Notes. الأرض نعبّرها مرة وحدة

$m \geq 1$:- m = number of Ind. parameters needed to control the mechanism.

$m = 0$: fixed structure (statically determ.)

$m < 1$: fixed structure (statically indeter.)

- **Degree of freedom of Gears**:-

→ gears contant Area has 2 Degrees of freedom.

Four bar mechanism

Granshof's Theory :-

S = length of the shortest l
L = 〃 〃 〃 longest link
P, q = 〃 〃 〃 other two links

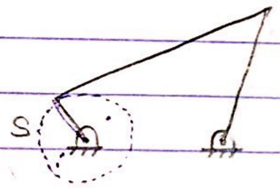
$S+L$ ■ $P+q$

$S+L < P+q$
At least one link can make a complete Revolution.
(Granshof's mechanism)

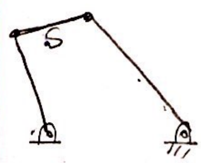
$S+L = P+q$
Change - point mechanism
• عند نقطة معينة كل الاضلاع بتصير على استقامة واحدة

$S+L > P+q$
Triple Rocker mechanism
→ no link can make a complete Revo.

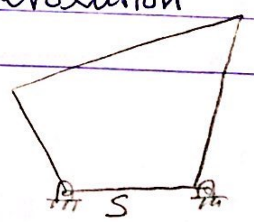
1- Cranck - Rocker . mech
S → one of the side links (make a rev)
All other link will oscillates



2- Double - Rocker . mech
S → the coupler
It will make a revalution
the other two links will oscillate



3- Double - Cranck . mech
S → the fixed Ground
All links make complete Revalution



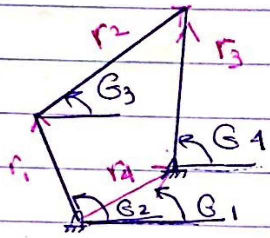
How to make position Analysis of 4-bar mechanism:-

1st Meth :- Graphical Method, using cosine and sine law.

2nd Meth :- Analytical Approach (complex numbers)

- نعبر عن كل link بمتجه بصيغة $e^{i\theta}$ حيث L هو طول ال link و θ هي زاوية ميلانه مع المحور x الموجب
- نستعمل ال Loops بحيث نبدأ من نقطة ونعود إليها ونحصل على معادلتين من كل loop، كل معادلة فيها Real part و Imaginary part

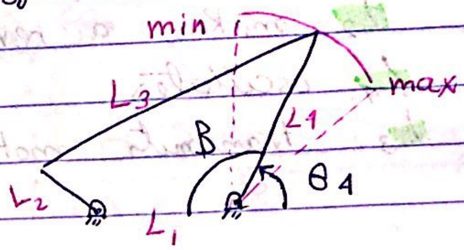
Ex:



$$r_1 + r_2 - r_3 - r_4 = 0$$

$$L_1(\cos\theta_2 + \sin\theta_2 \hat{j}) + L_2(\cos\theta_3 + \sin\theta_3 \hat{j}) - L_3(\cos\theta_4 + \sin\theta_4 \hat{j}) - L_4(\cos\theta_1 + \sin\theta_1 \hat{j}) = 0$$

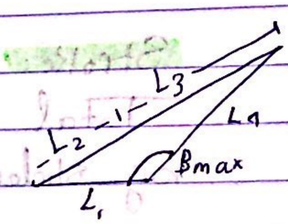
Range of Motion of the rocker in crank-rocker mechanism :- (Dead points of the rocker)



θ_4 is min
when β is max

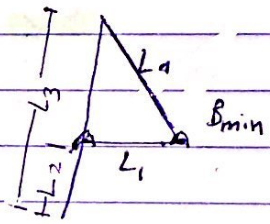
$\theta_4 + \beta = 180$

$$\beta_{max} = \cos^{-1}\left(\frac{L_1^2 + L_4^2 - (L_2+L_3)^2}{2L_1L_4}\right)$$



θ_4 is max
when β is min

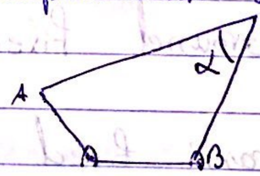
$$\beta_{min} = \cos^{-1}\left(\frac{L_1^2 + L_4^2 - (L_3-L_2)^2}{2L_1L_4}\right)$$



Transmission Angle : Between coupler link and output link

$|\alpha - 90^\circ| \leq 50^\circ$

$40^\circ \leq \alpha \leq 140^\circ$



$$\cos^{-1}\left(\frac{L_3^2 + L_4^2 - (AB)^2}{2L_3L_4}\right) = \alpha$$

→ max if AB is maximum → $AB = L_1 + L_2$

→ min if AB is minimum → $AB = L_1 - L_2$

* Slider crank mechanism

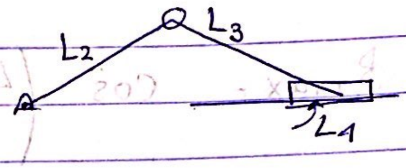
L_2: makes a rev

L_4: oscillates

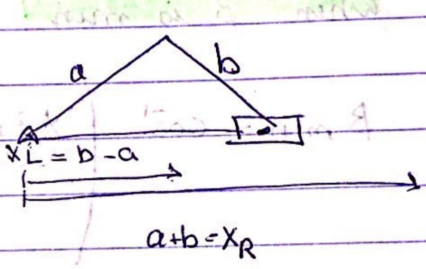
L_3: transmits motion from crank to the slider

Stroke:

Total distance Travelled by slider



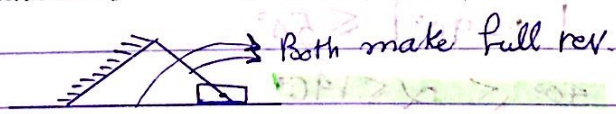
Stroke $= X_R - X_L = (a+b) - (b-a) = 2a$



* Types of slider crank mechanism :-

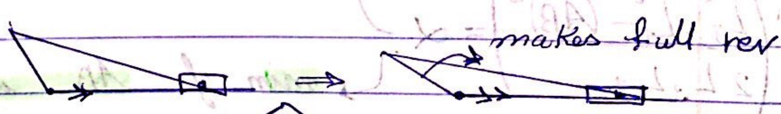
1- Ground fixed: Ground fixed

2- Crank fixed

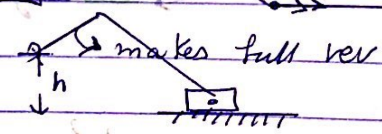


3- Connecting rod fixed

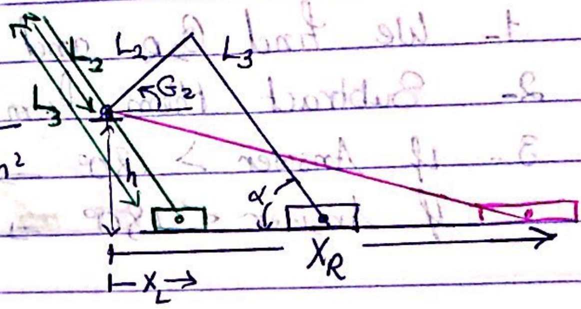
4- Slider fixed



5- offset slider crank



Offset slider :- (stroke)



$$\text{Stroke} = X_R - X_L = \sqrt{(L_2+L_3)^2 - h^2} - \sqrt{(L_3-L_2)^2 - h^2}$$

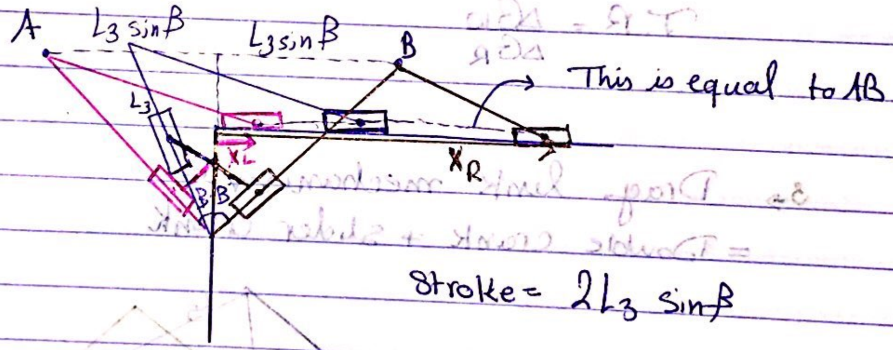
Quick Return mechanisms :-

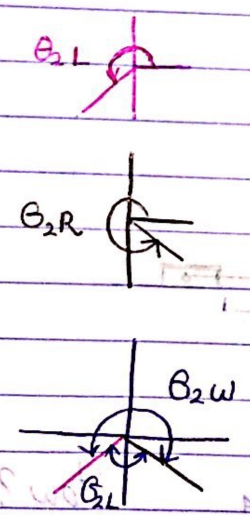
Working Speed < Return Speed

- But the crank has a constant Angular Velocity.

$$\text{Time ratio} = \frac{\Delta t \text{ working}}{\Delta t \text{ return}} \quad \text{always} > 1$$

1 → Crank shaper mechanism





$$\text{Stroke} = 2L_3 \sin\beta$$

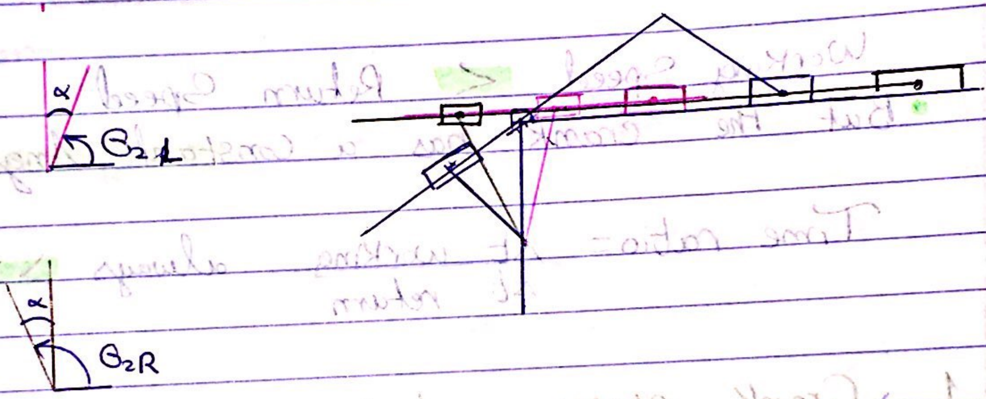
$$\omega_c = \frac{\Delta\theta \text{ return}}{\Delta t \text{ return}} = \frac{\Delta\theta \text{ work}}{\Delta t \text{ work}}$$

$$T.R = \frac{\Delta\theta_w}{\Delta\theta_R}$$

How to find $\Delta\theta_W$ and $\Delta\theta_R$:-

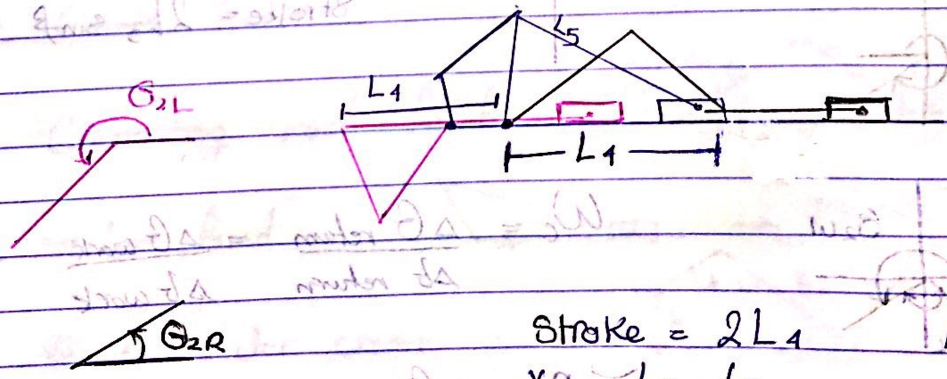
1- We find θ_{2R} and θ_{2L} (Remember θ_2 is Angle between X^+ and crank)
2- Subtract them from each other
3- If Answer $> 180^\circ \Rightarrow \Delta\theta_W$
If Answer $< 180^\circ \Rightarrow \Delta\theta_R$

2→ Whitworth mechanism



$$T.R = \frac{\Delta\theta_W}{\Delta\theta_R}$$

3→ Drag link mechanism :-
= Double crank + slider crank

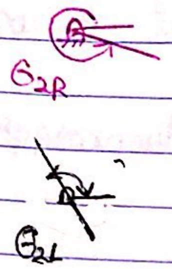


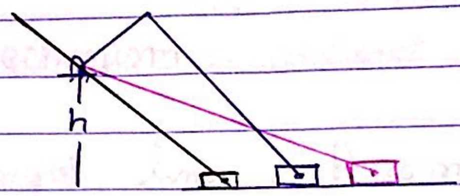
Stroke = $2L_4$ How?

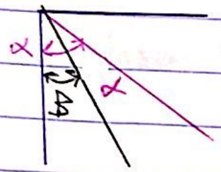
$X_R = L_4 + L_5$

$X_L = L_5 - L_4$ $\Rightarrow L_4 + \cancel{L_5} - \cancel{L_5} + L_4$
$= 2L_4$

4 → offset slider crank mechanism :-







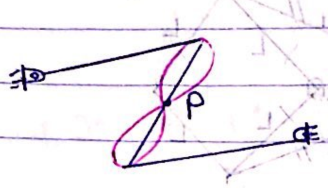
$\theta_{2R} = 270 + \alpha$
$\theta_{2L} = 90 + \beta$

$\Delta\theta = 180 + (\alpha - \beta)$ so $\Delta\theta_w$
$\Delta\theta_r = 360 - \Delta\theta_w$

Extra and Important mechanisms:-

Straight line mechanism :-

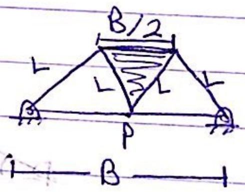
Approximate straight line , P is moving



P is closer to the longer link

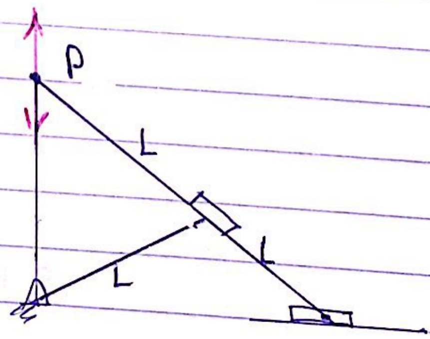
Robert mechanism :-

Approximate straight line , P is moving



Scott - Russell mechanism :-

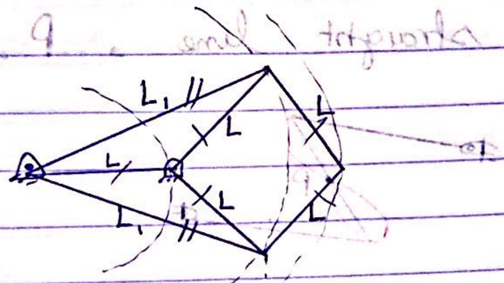
Exact straight line
P moves up-down



Scanned with CamScanner

Peaucelliers mechanism:

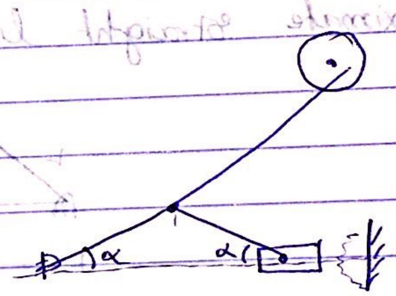
Exact straight line



Pantograph mechanism : Shape scale

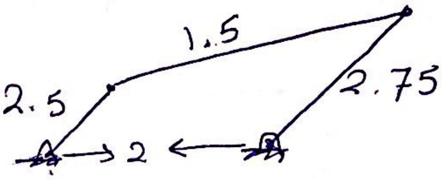
Generall well mechanism

Toggle mechanism



Examples

Ex1: Mechanism is ?



$S = 1.5 \Rightarrow$ The coupler

$L = 2.75$

$P = 2.5$

$q = 2$

$S+L \overset{?}{\lessgtr} p+q \Rightarrow 4.25 < 4.5$

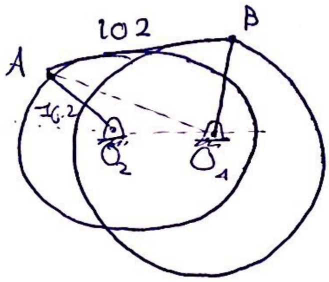
$\therefore$ It's a Double- Rocker mechanism

Ex2: $O_2A = 76.2$

$AB = 102$

$O_4\cancel{B} = 127$

Max of O_2O_4 ?



you have to find q, P, L

$q = 76.2$ (Drag link mechanism) = Double crank so $O_2O_4 = S$

$L = 127$

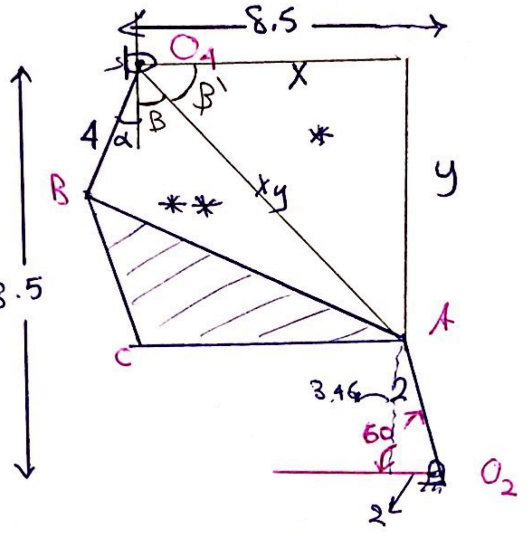
$P = 102$

$S + L \leq P + q$

$S + 127 \leq 102 + 76.2$

$S \leq 178.2 - 127 \Rightarrow S \leq 51.2$

Find the Angular Position of link 4 when link 2 is at 60°



$O_4B = 4$
$BC = 4$
$CA = 6$
$AO_2 = 4$
$AB = 8$

$\cos 60 \times 4 = 2$
$\sin 60 \times 4 =$

Angular position of link 4 with vertical is α

$\beta' + \beta = 90$

In Triangle *

$X = 8.5 - (2) = 6.5$

$y = 8.5 - (3.464) = 5.036$ $\qquad xy = \sqrt{y^2 + x^2} = 8.22$

so $\tan\beta' = \frac{y}{X} \Rightarrow \beta' = 37.76$

and $\beta = 52.233$

→ Now using cosine law in Triangle **

$8^2 = 4^2 + (8.22)^2 - (2)(4)(8.22)\cos(\alpha+\beta)$

So $\alpha + \beta = 72.688$

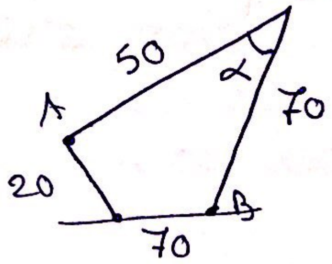
and $\boxed{\alpha = 20.45}$

<u>Example</u> :-

max α

min α

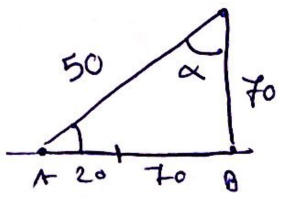
$\underline{\alpha \text{ is max}} \rightarrow AB$ is max



using cosine law

$$(20+70)^2 = (50)^2 + (70)^2 - (2)(50)(70)\cos\alpha$$

max $\boxed{\alpha_1 = 95.739}$ crank $\theta = 180$



α is min $\rightarrow AB$ is min

$$(50)^2 = (50)^2 + (70)^2 - (2)(50)(70)\cos\alpha$$

min $\boxed{\alpha_2 = 45.57}$ crank $\theta = 0$