In the linkage shown below, locate all of the instant centers.



## Problem 4.4

Find all of the instant centers of velocity for the mechanism shown below.



Locate all of the instant centers in the mechanism shown below. If link 2 is turning CW at the rate of 60 rad/s, determine the linear velocity of points C and E using instant centers.



Velocity Analysis

The two points of interest are on link 3. To find the angular velocity of link 3, use  $I_{13}$  and  $I_{23}$ . Then

$$v_{I_{23}} = 1\omega_2 \times \eta_{23/I_{12}} = 1\omega_3 \times \eta_{23/I_{13}}$$

Therefore,

$$|\mathbf{\omega}_3| = |\mathbf{\omega}_2| \frac{|\mathbf{v}_{123}/\mathbf{v}_1|}{|\mathbf{v}_{123}/\mathbf{v}_{13}|} = 60 \frac{1.2}{4.07} = 17.7 \text{ rad / s}$$

Because the instant center  $I_{23}$  lies between  $I_{12}$  and  $I_{13}$ ,  ${}^1\omega_3$  is in the opposite direction of  ${}^1\omega_2$ . Therefore,  ${}^1\omega_3$  is counterclockwise.

Then,

$$|v_{C_3} = |\omega_3 \times v_{C/I_{13}} \Rightarrow ||v_{C_3}| = ||\omega_3||v_{C/I_{13}}| = 17.7 \cdot 2.11 = 37.3 \text{ in / s}$$

and

$$|v_{E_3} = |\omega_3 \times r_{E/I_{13}} \Rightarrow ||v_{E_3}| = ||\omega_3|r_{E/I_{13}}| = 17.7 \cdot 3.25 = 57.5 \text{ in/s}$$

The directions for the velocity vectors are shown in the drawing.



Locate all of the instant centers in the mechanism shown below. If the cam (link 2) is turning CW at the rate of 900 rpm, determine the linear velocity of the follower using instant centers.



## Instant Centers



#### **Velocity of the Follower**

Convert the angular velocity from "rpm" to "rad/s"

$$^{1}\omega_{2} = 900 \ rpm = \frac{900(2\pi)}{60 \, \text{sec}} = 94.25 \ rad / s \ CW$$

At the point  $I_{23}$  the linear velocity of follower and cam is same.

$${}^{1}\mathbf{v}_{I_{23}} = {}^{1}\mathbf{v}_{A_{2}} + {}^{1}\mathbf{v}_{I_{23}/A_{2}} = 0 + {}^{1}\omega_{2} \times \mathbf{r}_{I_{23}/A} = (94.25 \ rad \ / \ s)(0.82 \ in) = 77.285 \ in \ / \ s \quad Down$$

#### Problem 4.7

Locate all of the instant centers in the mechanism shown below. If link 2 is turning CW at the rate of 36 rad/s, determine the linear velocity of point  $B_4$  by use of instant centers. Determine the angular velocity of link 4 in rad/s and indicate the direction. Points C and E have the same vertical coordinate, and points A and C have the same horizontal coordinate.



#### Solution:

Find all instant centers and linear velocity of point B2.

$$|\mathbf{w}_{B_2} = |\omega_2 \times \mathbf{r}_{B_2/A_2} \Rightarrow |\mathbf{w}_{B_2}| = |\omega_2| \cdot |\mathbf{r}_{B_2/A_2}| = 36 \cdot 1.1 = 39.6 \text{ in / s}$$

Using rotating radius method,

$$w_{B_4} = 32.5 \text{ in } / \text{s}$$

To calculate the angular velocity of link 4, we can use the relations between related instant centers.

$$|\omega_{2} \times \mathbf{r}_{l_{24}/l_{12}} = |\omega_{4} \times \mathbf{r}_{l_{24}/l_{14}}|$$
$$|\omega_{4}| = |\omega_{2}| \cdot \frac{|\mathbf{r}_{l_{24}/l_{12}}|}{|\mathbf{r}_{l_{24}/l_{14}}|} = 36 \cdot \frac{1.283}{2.186} = 21.1 \ rad \ / \ s$$

Therefore,

 $|\omega_4| = 21.1 \ rad / s \ CW$ 





Using the instant-center method, find angular velocity of link 6 if link 2 is rotating at 50 rpm CCW.

If  $\boldsymbol{\omega}_2 = 5$  rad/s CCW, find  $\boldsymbol{\omega}_5$  using instant centers.



## Solution:

Draw linkage to scale and find necessary instant centers  $(I_{12}, I_{15}, and I_{25})$ .

The relationshp between  ${}^{1}\omega_{2}$  and  ${}^{1}\omega_{5}$  is

$${}^{1}\omega_{2} \times n_{25}/I_{12} = {}^{1}\omega_{5} \times n_{25}/I_{15}$$
(1)

Solve Eq. (1) for  $1 \omega_3$ ,

$$|\mathbf{w}| = |\mathbf{w}| \cdot \frac{|\mathbf{r}_{125/I_{12}}|}{|\mathbf{r}_{125/I_{15}}|} = 5 \cdot \frac{1.83}{2.27} = 4.03 \text{ rad / s}$$

So,

$$\omega_5 = 4.03 \text{ rad/s CW}$$





If  $\omega_2 = 1$  rad/s CCW, find the velocity of point *A* on link 6 using the instant center method. Show  $v_{A6}$  on the drawing.



Solution:



Find necessary instant centers, i.e.  $I_{12}$ ,  $I_{16}$ , and  $I_{26}$ , and the velocity of point D as

If the velocity of  $A_2$  is 10 in/s to the right, find  $\omega_6$  using instant centers.



## Solution:

Find necessary instant centers as shown in the sketch above, i.e.  $I_{12}$ ,  $I_{16}$ , and  $I_{26}$ . All points in link 2 have the same velocity; therefore,

$$\mathbf{w}_{A_2} = \mathbf{w}_{A_2} = \mathbf{w}_{A_2}$$

Using the rotating radius method,

$$w_{D_6} = w_{D_6/E_6} = 13.2$$
 in / s

Now,

$$|\mathbf{w}_{D_6/E_6} = |\omega_6 \times \mathbf{r}_{D_6/E_6} \Longrightarrow |\omega_6| = \frac{|\mathbf{w}_{D_6/E_6}|}{|\mathbf{r}_{D_6/E_6}|} = \frac{13.2}{2.25} = 5.87 \,\mathrm{rad}\,/\mathrm{s}$$

Therefore,

$$|\omega_0| = 5.87 \text{ rad} / \text{s CW}$$



Crank 2 of the push-link mechanism shown in the figure is driven at  $\omega_2 = 60$  rad/s (CW). Find the velocity of points *B* and *C* and the angular velocity of links 3 and 4 using the instant center method.



Solution:

Find all instant centers and velocity of point A

$$|\mathbf{v}_{A_2} = |\mathbf{\omega}_2 \times \mathbf{r}_{A_2/O_2} \Rightarrow |\mathbf{v}_{A_2}| = |\mathbf{\omega}_2| \times |\mathbf{r}_{A_2/O_2}| = 60 \cdot 0.015 = 0.9 \ m \ / \ s$$

Using rotating radius method,

$$w_{B_3} = 1.15 \text{ m/s}$$