Kinematic Complex Number Analysis

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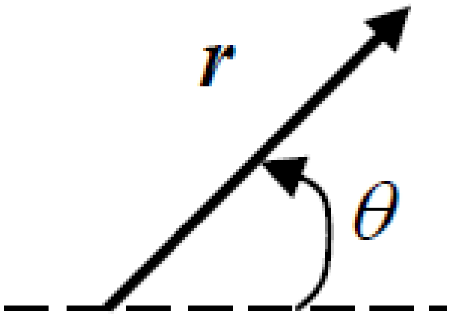
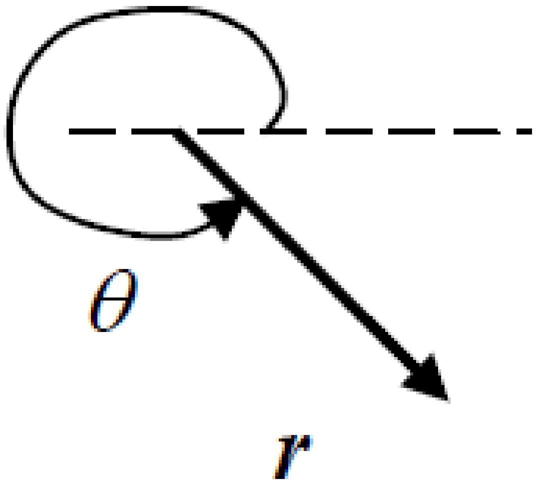
Vector Analysis

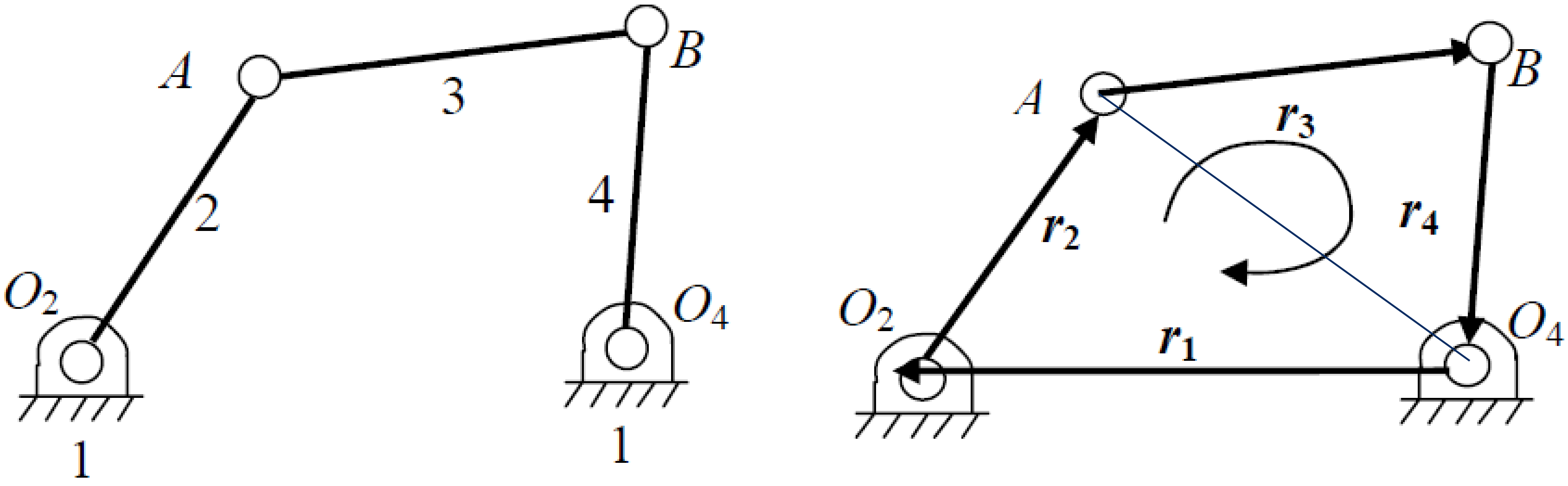
$$r = r_x + i\, r_y = r\,(\cos\theta + i\,\sin\theta) = r\, e^{i\theta}$$





Vector Loop-Closure Equation

Four Bar Linkage

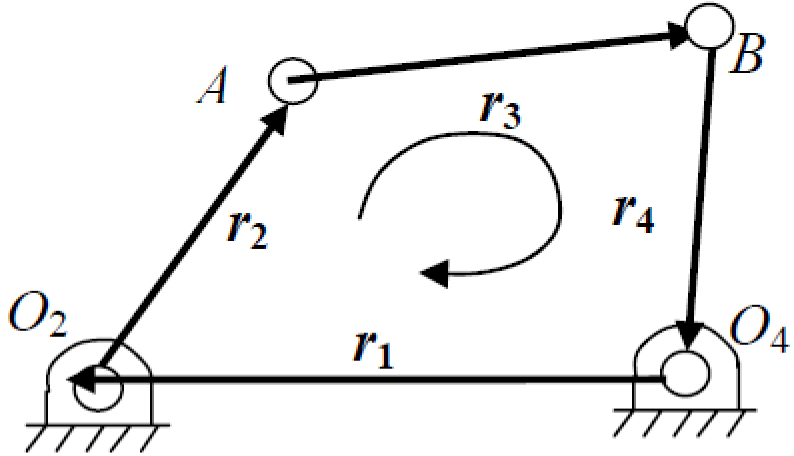


Vector Loop-Closure Equation
Four Bar Linkage
Position Vector:

$$r_1 + r_2 + r_3 + r_4 = 0$$

$$r_1 e^{i\theta_1} + r_2 e^{i\theta_2} + r_3 e^{i\theta_3} + r_4 e^{i\theta_4} = 0 \ (1)$$

$$r_1(\cos\theta_1 + i\sin\theta_1) + r_2(\cos\theta_2 + i\sin\theta_2) + r_3(\cos\theta_3 + i\sin\theta_3) + r_4(\cos\theta_4 + i\sin\theta_4) = 0$$



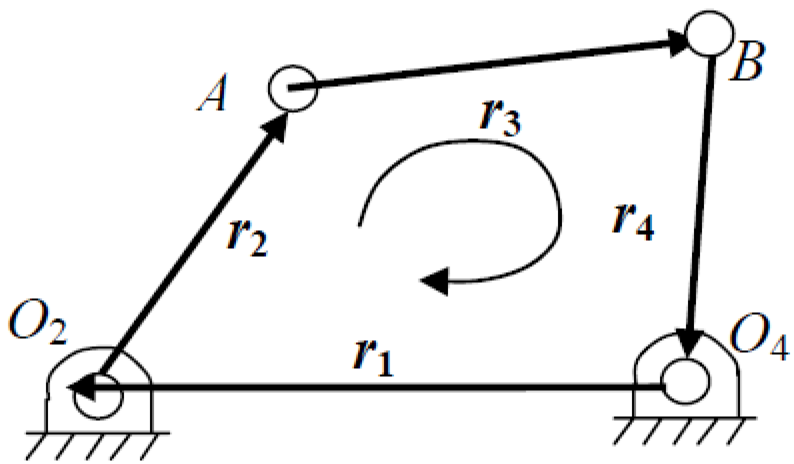
Vector Loop-Closure Equation
Four Bar Linkage

Position Vector:

Real: $r_1\cos\theta_1 + r_2\cos\theta_2 + r_3\cos\theta_3 + r_4\cos\theta_4 = 0$

Imag: $r_1\sin\theta_1 + r_2\sin\theta_2 + r_3\sin\theta_3 + r_4\sin\theta_4 = 0$

Given: $r_1, r_2, r_3, r_4, \theta_1, \theta_2$ Find: θ_3, θ_4



Vector Loop-Closure Equation

Four Bar Linkage

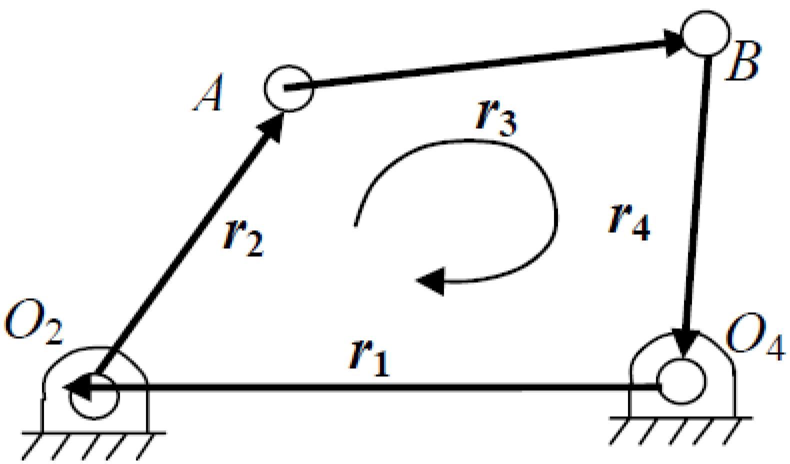
Velocity Analysis: $$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \dot{\boldsymbol{r}} = (\dot{r} + ir\omega)\,\mathrm{e}^{i\theta}$$

$$\dot{r}_1 + (\dot{r}_2 + ir_2\omega_2)e^{i\theta_2} + (\dot{r}_3 + ir_3\omega_3)e^{i\theta_3} + (\dot{r}_4 + ir_4\omega_4)e^{i\theta_4} = 0$$

Real: $-r_2\omega_2\sin\theta_2 - r_3\omega_3\sin\theta_3 - r_4\omega_4\sin\theta_4 = 0$

Imag: $r_2\omega_2\cos\theta_2 + r_3\omega_3\cos\theta_3 + r_4\omega_4\cos\theta_4 = 0$

Given: All r's and θ's, and ω_2 Find: ω_3, ω_4



Vector Loop-Closure Equation
Four Bar Linkage

Acceleration Analysis:

$$\frac{d\dot{\boldsymbol{r}}}{dt} = \ddot{\boldsymbol{r}} = (\ddot{r} + i2\dot{r}\,\omega + ir\alpha - r\omega^2)e^{i\theta}$$

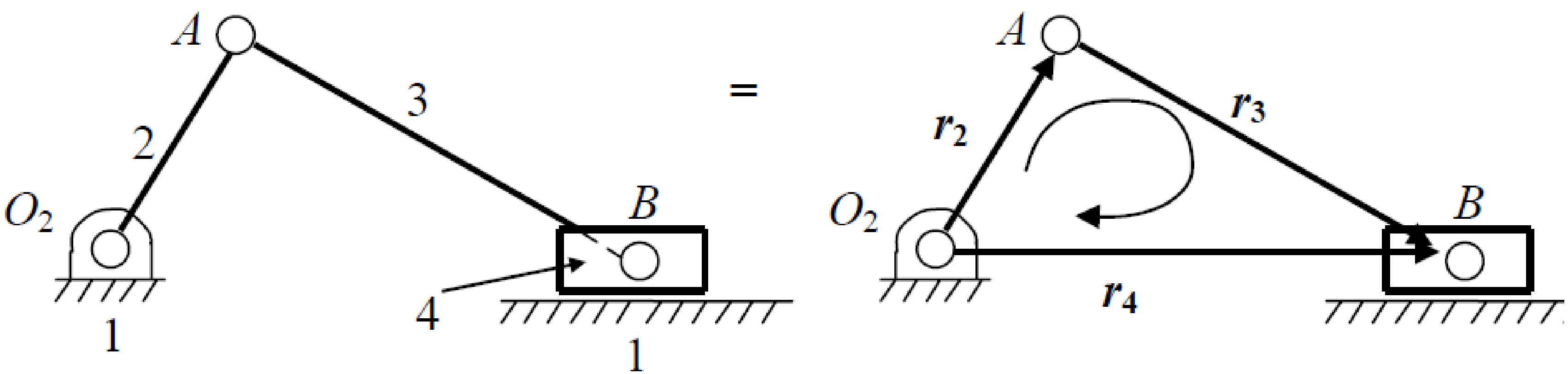
$$\ddot{r}_1 + (\ddot{r}_2 + i2\dot{r}_2\,\omega_2 + ir_2\alpha_2 - r_2\,\omega_2^2)\,e^{i\theta_2} + (\ddot{r}_3 + i2\dot{r}_3\,\omega_3 + ir_3\alpha_3 - r_3\,\omega_3^2)\,e^{i\theta_3}$$
$$+ (\ddot{r}_4 + i2\dot{r}_4\,\omega_4 + ir_4\alpha_4 - r_4\,\omega_4^2)\,e^{i\theta_4} + = 0$$

Real: $-r_2\alpha_2\sin\theta_2 - r_2\,\omega_2^2\cos\theta_2 - r_3\alpha_3\sin\theta_3 - r_3\,\omega_3^2\cos\theta_3 - r_4\alpha_4\sin\theta_4 - r_4\,\omega_4^2\cos\theta_4 = 0$

Imag: $r_2\alpha_2\cos\theta_2 - r_2\,\omega_2^2\sin\theta_2 + r_3\alpha_3\cos\theta_3 - r_3\,\omega_3^2\sin\theta_3 + r_4\alpha_4\cos\theta_4 - r_4\,\omega_4^2\sin\theta_4 = 0$

Given: All r's, θ's, and ω's, and α_2 Find: α_3, α_4

Vector Loop-Closure Equation
Slider Crank Mechanism



Vector Loop-Closure Equation

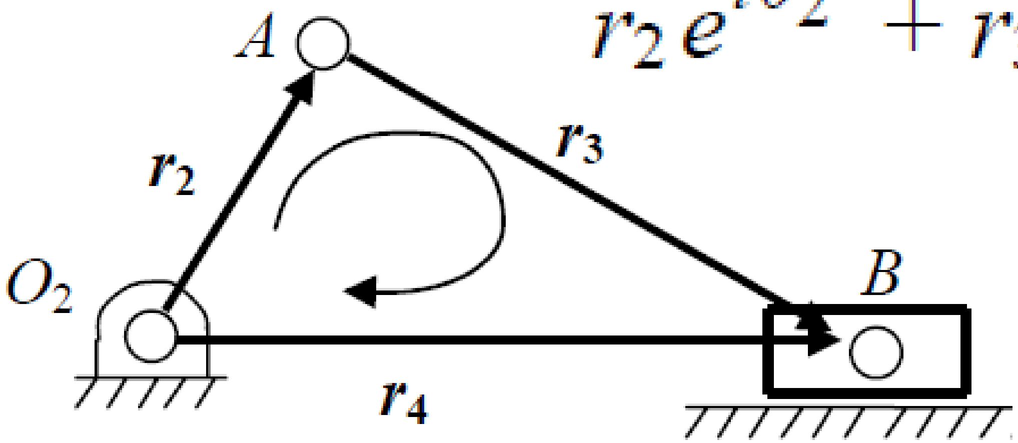
Slider Crank Mechanism

Position Vector:

$r_2 + r_3 - r_4 = 0$

$$r_2 e^{i\theta_2} + r_3 e^{i\theta_3} - r_4 e^{i\theta_4} = 0$$

$$r_2 e^{i\theta_2} + r_3 e^{i\theta_3} - r_4 = 0$$



Vector Loop-Closure Equation

Slider Crank Mechanism

Position Vector:

$r_2 + r_3 - r_4 = 0$

$$r_2(\cos\theta_2 + i\sin\theta_2) + r_3(\cos\theta_3 + i\sin\theta_3) - r_4 = 0$$

$$\text{Real:} \quad r_2\cos\theta_2 + r_3\cos\theta_3 - r_4 = 0$$

$$\text{Imag:} \quad r_2\sin\theta_2 + r_3\sin\theta_3 = 0$$

Given: r_2, r_3 and θ_2 Find: θ_3, r_4

Vector Loop-Closure Equation

Slider Crank Mechanism

Velocity Analysis:

$r_2 + r_3 - r_4 = 0$

$$(\dot{r}_2 + i r_2 \omega_2) e^{i\theta_2} + (\dot{r}_3 + i r_3 \omega_3) e^{i\theta_3} - \dot{r}_4 = 0$$

Real: $-r_2\omega_2 \sin\theta_2 - r_3\omega_3 \sin\theta_3 - \dot{r}_4 = 0$

Imag: $r_2\omega_2 \cos\theta_2 + r_3\omega_3 \cos\theta_3 = 0$

Given: All r's and θ's, and ω_2 Find: ω_3, $\dot{r}_4$

Vector Loop-Closure Equation

Slider Crank Mechanism

Acceleration Analysis:

$$(\dot{r}_2 + i r_2 \omega_2) e^{i\theta_2} + (\dot{r}_3 + i r_3 \omega_3) e^{i\theta_3} - \dot{r}_4 = 0$$

$$(\ddot{r}_2 + i2\dot{r}_2\omega_2 + i r_2 \alpha_2 - r_2 \omega_2^2) e^{i\theta_2}$$

$$+ (\ddot{r}_3 + i2\dot{r}_3\omega_3 + i r_3 \alpha_3 - r_3 \omega_3^2) e^{i\theta_3} - \ddot{r}_4 = 0$$

Real: $-r_2\alpha_2 \sin\theta_2 - r_2\omega_2^2\cos\theta_2 - r_3\alpha_3\sin\theta_3 - r_3\omega_3^2\cos\theta_3 - \ddot{r}_4 = 0$

Imag: $r_2\alpha_2\cos\theta_2 - r_2\omega_2^2\sin\theta_2 + r_3\alpha_3\cos\theta_3 - r_3\omega_3^2\sin\theta_3 = 0$

Given: All r's, θ's, and ω's, and α_2 Find: α_3, $\ddot{r}_4$

Example 15.3

If the crank and the connecting rod are 300 mm and 1000 mm long respectively and the crank rotates at a constant speed of 200 rpm, determine:

1. *The crank angle at which the maximum velocity occurs*
2. *Maximum velocity of the piston.*

Example 15.4

The crank and connecting rod of a steam engine are 0.3 m and 1.5 m in length. The crank rotates at 180 rpm clockwise. Determine the velocity and acceleration of the piston when the crank is at 40 degrees from the IDC position. Also determine the position of the crank for zero acceleration of the piston.

Example 15.5

In a slider crank mechanism, the length of the crank and connecting rod are 150 mm and 600 mm respectively. The crank position is 60° from IDC. The crank shaft speed is 450 rpm cw. Using analytical method, determine:

1. *Velocity and acceleration of the slider*
2. *Angular velocity and angular acceleration of the connecting rod*