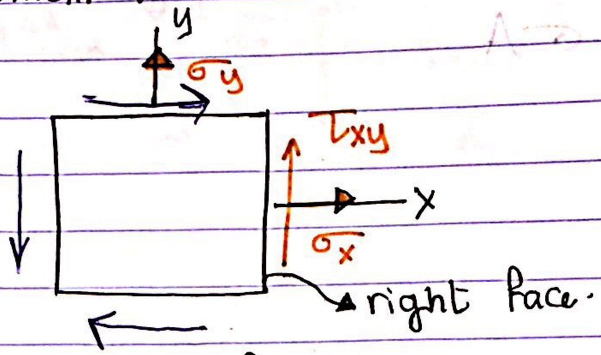
Chapter 9: Stress Transformation

9.1- plane stress transformation

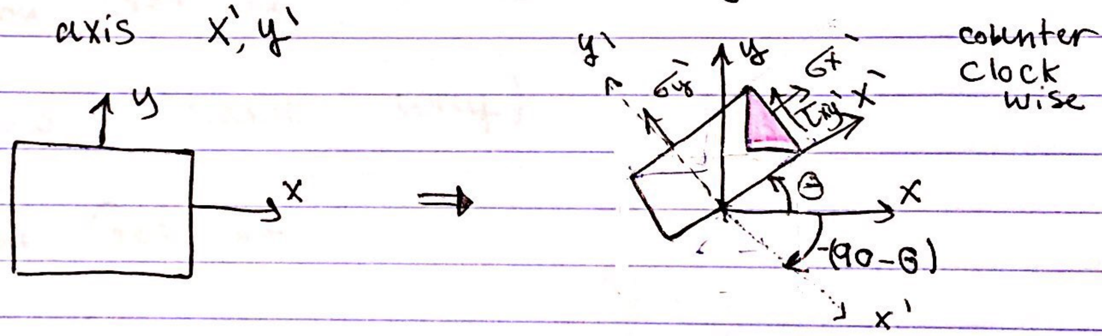
σ_x , σ_y , τ_{xy} acts on four faces of the element.

▶ Positive sign Convention



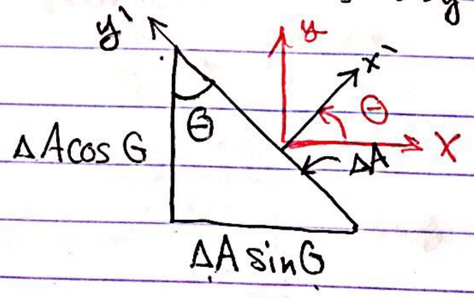
σ_x : Tension
σ_y : Tension
τ_{xy} : upward on the right face.

→ If The element oriented θ Degrees counterclock- or clockwise a new σ_x σ_y, τ_{xy} are formed on axis x', y'

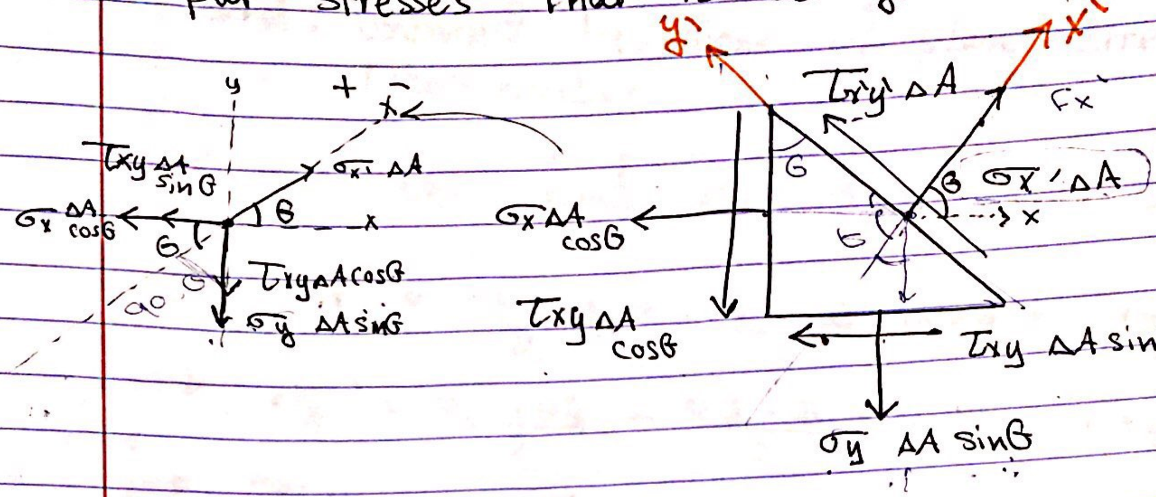


• To find $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$:-

→ Take an element to find $\sigma_{x'}$, $\tau_{x'y'}$ (pink Region)



• put stresses That is acting on the element



$\Sigma F_{x'} = 0$

$\sigma_{x'} \Delta A - (\tau_{xy} \Delta A \sin\theta) \cos\theta - (\sigma_x \Delta A \cos\theta) \cos\theta$

$- (\tau_{xy} \Delta A \cos\theta)(\sin\theta) - (\sigma_y \Delta A \sin\theta)(\sin\theta) = 0$

You Get $\sigma_{x'}$

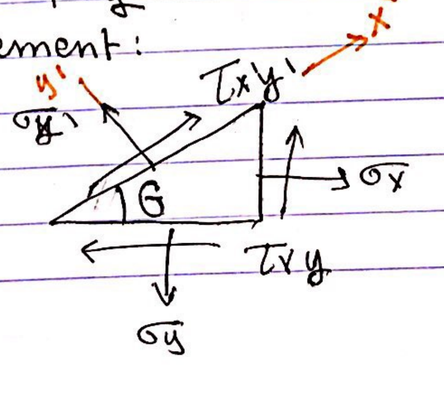
$\Sigma F_{y'} = 0$ (same way)

You Get $\tau_{x'y'}$

• To find $\sigma_{y'}$:

use $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$

or use another element:



9.2: General Equations of plane-stress Transformation.

* You can find $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$ usin Eqs.

→ $$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

→ $$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

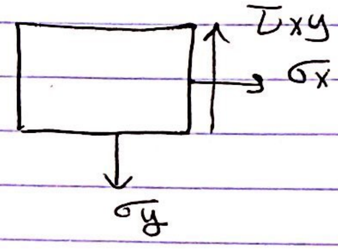
→ $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

◄ نعوّض θ مع الإشارة وهي موجبة عندما تكون من X لـ X'

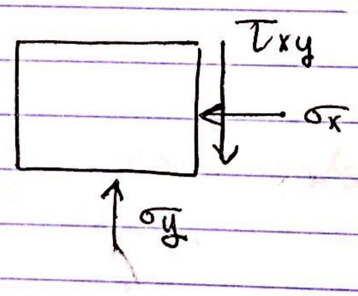
(Counter clockwise يعني)

◄ نعوّض σ_x σ_y τ_{xy} مع الإشارة

$+\sigma_x$, $+\sigma_y$, $+\tau_{xy}$: هنا



$-\sigma_x$, $-\sigma_y$, $-\tau_{xy}$: هنا



9.3: Principal stresses and Maximum In-plane Shear stress.

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

θ_p: is θ Between x and x' where x' is a principal axis.

$$\sigma_{1,2} \text{ (Maximum and Minimum stresses)} = \underbrace{\frac{\sigma_x + \sigma_y}{2}}_{\sigma_{avg}} \pm \underbrace{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}_{R}$$

on principal axis $\tau_{x'y'} = 0$

To find axes That has the max shear stress

$$\tan 2\theta_s = \frac{-\frac{(\sigma_x - \sigma_y)}{2}}{\tau_{xy}}$$

Angle Between principal axis and Max Shear stress axis = 45°

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$ Average Stress acts in This case.

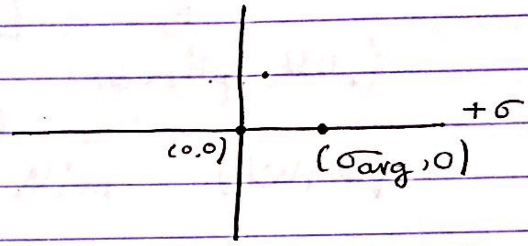
$$\tau_{x'y'\,max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

9.4: Mohr's Circle - Plane stress

- To Draw Mohr's Circle :-

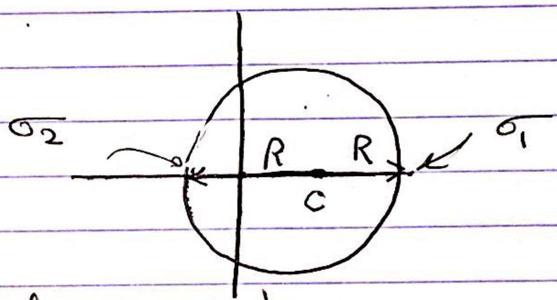
① esablish the Center $(\sigma_{avg}, 0)$

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$ with signs.



② find R and Draw the circle:

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$



③ Find σ_1, σ_2 (principal stresses)

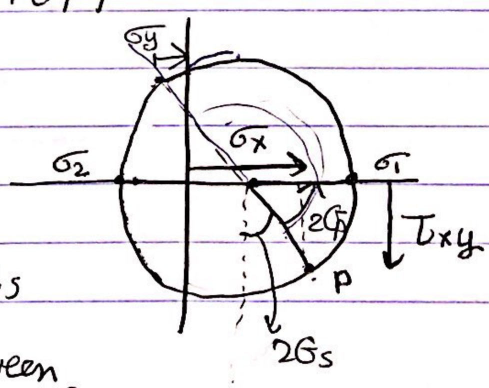
$\sigma_1, \sigma_2 =$

$+\sigma_1 = R + \sigma_{avg}$

$\sigma_2 = -R + \sigma_{avg}$

④ find Point of start (ref)

P (σ_x, τ_{xy})



at $P \Rightarrow \theta = 0$

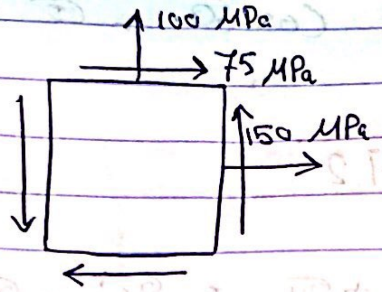
and we can Get $2\theta_p$

↳ Angle Between P and X-axis

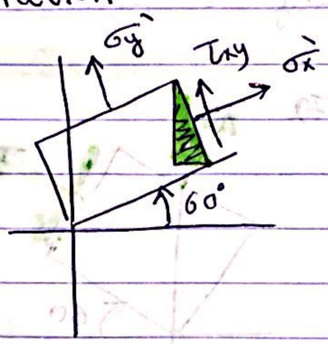
and $2\theta_s$ is the Angle Between P and y-axis $(90 - 2\theta_{p_1})$

9.11 oriented 60° counter clockwise

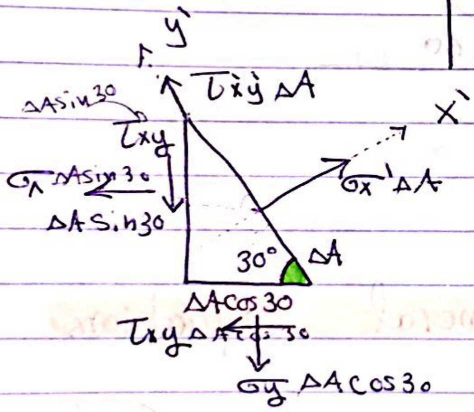
Method 1 :- 9.1 using an element



→ after orientation



(counter clockwise)



$\Sigma F_{x'} = 0$:- $\sigma_{x'} \cancel{\Delta A} - \sigma_x (\cancel{\Delta A} \sin 30) \cos 60$

$- \tau_{xy} \cancel{\Delta A} \sin 30 \cos 30$

$- \tau_{xy} \cancel{\Delta A} \cos 30 \cos 60 - \sigma_y \cancel{\Delta A} \cos 60 \cos 30$

$= 0$

$\sigma_{x'} - \sigma_x \sin 30 \cos 60 - \tau_{xy} \sin 30 \cos 30$

$- \tau_{xy} \cos 30 \cos 60 - \sigma_y \cos 60 \cos 30 = 0$

$\sigma_{x'} - (150) \sin 30 \cos 60 - 75 \sin 30 \cos 30 - 150$

$\cos 30 \cos 60 - 100 \cos 60 \cos 30 = 0$

$\boxed{\sigma_{x'} = 178.229}$

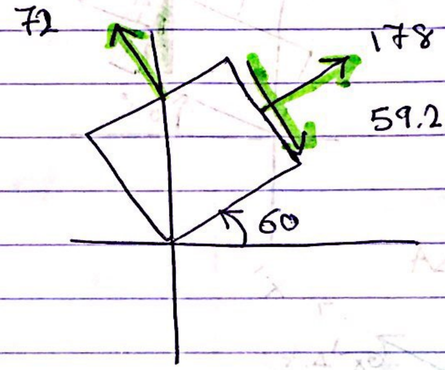
$\Sigma F_{y'} = 0$

$$\tau_{x'y'} \Delta A - \overset{75}{\tau_{xy}} \Delta A \sin 30 \sin 30 + \overset{150}{\sigma_x} \Delta A \sin 30 \cos 30$$
$$- \underset{100}{\sigma_y} \Delta A \cos 30 \cos 60 + \tau_{xy} \Delta A \cos 30 \cos 30 = 0$$

$$\boxed{\tau_{x'y'} = -59.2}$$

→ Using: $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$

$$\boxed{\sigma_{y'} = 71.47 \simeq 72}$$



Method 2: 9.2 General equations

$\sigma_x = +150$ MPa $\theta = +60$ (c.c)

$\sigma_y = +100$ MPa

$\tau_{xy} = +75$ MPa

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\boxed{\sigma_{x'} = 177 \text{ MPa}}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\boxed{\sigma_{y'} = 72.5 \text{ MPa}}$$

$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \quad \boxed{= -59.2 \text{ MPa}}$$

Method 3. Mohr's Circle

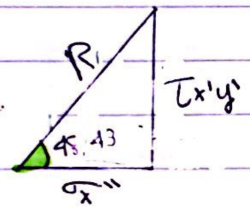
1- Center : $\sigma_{avg} = \frac{100 + 150}{2} = 125$

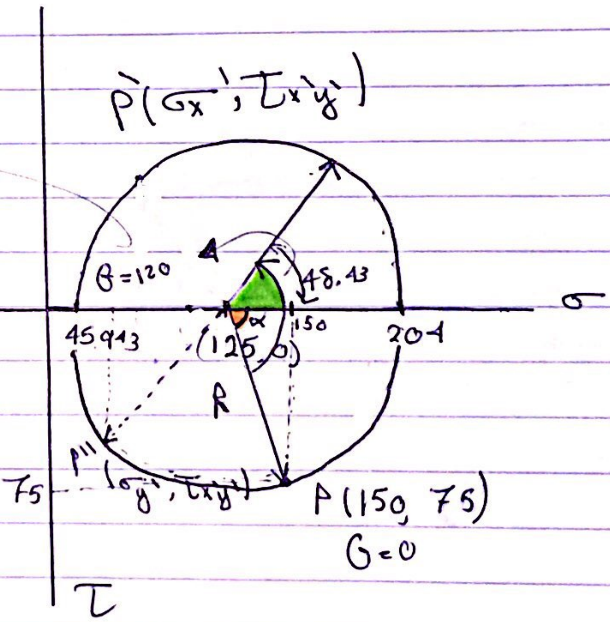
$\hookrightarrow (125, 0)$

2- $R = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 79.057$

3_ θ of orientation $= +60$ and Its 120 on the circle

4- find ref $(\theta = 0)$: $P(\sigma_x, \tau_{xy})$





$\sin 48.4 = \frac{\tau_{x'y'}}{R}$

$\tau_{x'y'} = 59.146$

$\cos 48.43 = \frac{\sigma_x''}{R}$

$\sin \alpha = \frac{75}{79.075} =$

$\alpha = 71.5258$ Between P and σ axis

$\sigma_x' = \sigma_x'' + 125$

$= 52.457 + 125 = 177.45$

To find σ_y' . Take p''

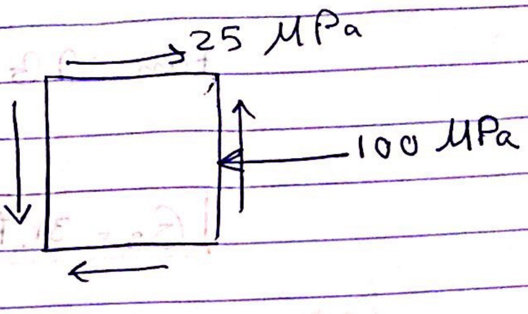
and $\sigma_y' = 125 - 52.457 =$
$= 72.5$

9.19.

Method 1: 9.3 equations

$\sigma_x = -100 \quad \sigma_y = 0$

$\tau_{xy} = 25$



1- principal stresses:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\sigma_x - \sigma_y/2} = \frac{25}{\frac{-100}{2}} = \frac{25}{-50}$$

$\boxed{\theta_p = -13.28}$ (clockwise)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$= -50 \pm 55.9$

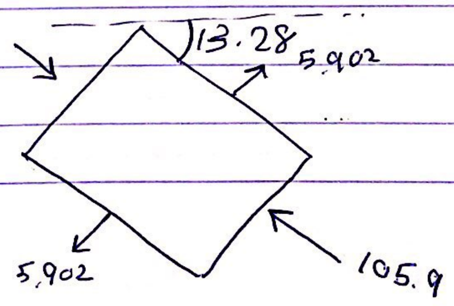
$\sigma_{1,2} = -105.9 \ / \ 5.902$

find $\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_p + \tau_{xy}\sin 2\theta_p$

$= -50 - (50)(0.894) + (25)(-0.447)$

$$\boxed{\begin{aligned}\sigma_x' &= -105.9 \\ \sigma_y' &= 5.902\end{aligned}}$$

$\tau_{xy} = 0$



Scanned with CamScanner

2- Max In-plane Shear Stress

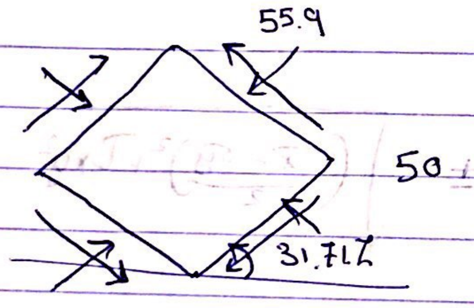
$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}} = \frac{100}{50} = 2$$

$\boxed{\theta_s = 31.717}$ Counter clock wise

$$\sigma = \sigma_{avg} = \frac{-100}{2} = -50 \text{ MPa}$$

and $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$\tau_{max} = \sqrt{\left(\frac{-100}{2}\right)^2 + (25)^2} = 55.9 \text{ MPa}$$



Method: Mohr's Circle

1- Center $\sigma_{avg} = -50$
↳ $(-50, 0)$

2- $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 55.9$

3- ref $\theta = 0$ is at $P(\sigma_x, \tau_{xy})$
$= P(-100, 25)$

4- find $2\theta_p \Rightarrow$ It's negative → clockwise

$\sin 2\theta_p = \frac{25}{55.9}$

$\sigma_{1,2} = -105.9 / 5.9$

$\tau_{xy} = 0$

$\theta_p = 13.28$

5- find $2\theta_s \Rightarrow$

$90 - 26.56 = 2\theta_s$

$\theta_s = 31.72$

$\tau_{max} = 55.9$