

Basic Definitions :-

1- Classical: $P(A) = \frac{\text{num of elements of } A}{S}$

where A : an event
 S : Sample space

2- Relative Frequency $\Rightarrow P(A) = \lim_{n \rightarrow \infty} \frac{\text{num of trials } A \text{ occurs}}{\text{number of trials}}$

3- subjective

4- Axiomatic:-

$$P(A) \geq 0$$

$$P(S) = 1$$

$$P(A \cup B) = P(A) + P(B) \quad \rightarrow \quad \text{if } A \text{ \& } B \text{ are disjoint}$$

$$A \cap B = \emptyset$$

$$P(A^c) = 1 - P(A)$$

$$P(\emptyset) = 0$$

if A & B are not disjoint then:-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

if A & B & C ~ ~ ~ ~ :-

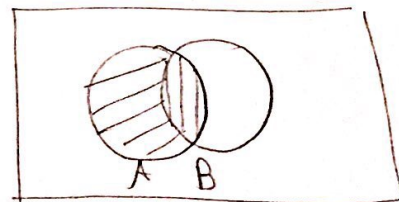
$$\rightarrow P(A \cap B) = 0, P(A \cap C) = 0, P(B \cap C) = 0$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Conditional probability :-

$P(A/B)$ means prob of A given B

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$



Bay's Theorem:

$$P(A|B) P(B) = P(B|A) P(A)$$

Chain rule:-

$$P(A \cap B) = P(A) P(B|A)$$

$$\begin{aligned} P(A \cap B \cap C) &= P(A) P(B|A) P(C|A, B) \\ &= P(B) P(A|B) P(C|A, B) \\ &= P(C) P(A|C) P(B|A, C) \end{aligned}$$

statically independent :- احتمالية تحدث لا تتأثر على الأخرى

$$P(A) = P(A|B) \quad \text{or} \quad P(B|A) = P(B) \quad \text{or} \quad P(A \cap B) = P(A)P(B)$$

for three events :-

$$\begin{aligned} P(A \cap B) &= P(A)P(B) \\ P(A \cap C) &= P(A)P(C) \\ P(B \cap C) &= P(B)P(C) \\ P(A \cap B \cap C) &= P(A)P(B)P(C) \end{aligned}$$

← يجب التحقق من كلهم

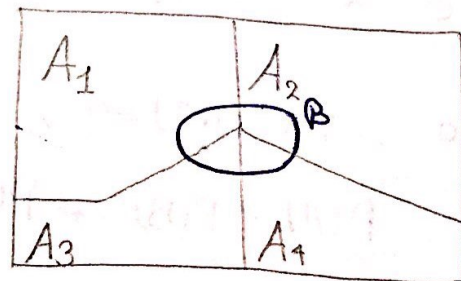
Morgan's law:-

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

Theorem of Total Probability :-

$$\begin{aligned} P(B) &= P(A_1) P(B|A_1) + P(A_2) P(B|A_2) \\ &+ P(A_3) P(B|A_3) + P(A_4) P(B|A_4) \end{aligned}$$



Counting Techniques:

→ Multiplication Rule :-

You have two events in sequence, the way to know how many ways you can arrange or perform the sequence is $n_1 \times n_2$

where n_1 : ways A can be performed
 n_2 : ways B can be performed

Experiment n : number of (floors, balls, trials)

k : number of trials

No replacement

Sample space = $n!$

order Imp

permutations

$$P_k^n = \frac{n!}{(n-k)!}$$

order Not Imp

combinations

$$\binom{n}{k} = \frac{n!}{(n-k)! k!}$$

Replacement is allowed

Sample space = n^k

$$N = n^k$$