Elementary statics:-

Questions: find linearization

Defs:-

- Sample Mean $\hat{\mu}_x = \frac{1}{n} \sum_{i=1}^{n} x_i$
- Sample Variance $\hat{\sigma_x}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \hat{\mu}_x)^2$
- Sample standard Deviation $\hat{\sigma_x} = \sqrt{\hat{\sigma_x}^2}$
- $\hat{\sigma_x}^2 =$ (also) $\frac{n \sum x_i^2 - (\sum x_i)^2}{n(n-1)}$

Regression Techniques:

- finding $y = \alpha x + \beta$ form (appropiate)

$\alpha = \frac{C_{xy}}{\sigma_x^2} = \sum xy - n \hat{\mu}_x \hat{\mu}_y / \sum x_i^2 - n \hat{\mu}_x^2$

$\beta = \hat{\mu}_y - \alpha \hat{\mu}_x$

- Sample correlation coefficient $\rho_{x,y} = \frac{C_{xy}}{\hat{\sigma_x} \hat{\sigma_y}}$

and $C_{xy} = n \sum x_i y_i - \sum x_i \sum y / n(n-1)$

if $y = Ae^{-Bx}$

Then Take ln for each side

$\ln y = \ln A - B \cancel{\ln e^{1}}(x)$

$= \underbrace{\ln A}_{\beta} - x \underbrace{B}_{\alpha}$

Fitting a polynomial by Method of least squares:

$$y = \beta_1 + \beta_2 x + \beta_3 x^2$$

$$\begin{bmatrix} n & \Sigma x_i & \Sigma x_i^2 \\ \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 \\ \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \\ \Sigma x_i^2 y_i \end{bmatrix}$$

- These matrices are used to find β_1, β_2, β_3

There are 4 Theorems $P(70+71)$

① $X_1, \ldots, X_n$ Gaussien R.V
Then if $y = c_1X_1 + \ldots + c_nX_n$

$$\mu_y = c_1\mu_1 + \ldots + c_n\mu_n$$
$$\sigma_y^2 = c_1^2\sigma_1^2 + \ldots + c_n^2\sigma_n^2 + 2c_1c_2\sigma_1\sigma_2\rho_{12} + \ldots$$

② $X_1, \ldots, X_n$ Ind ~ ~

Then if $y = c_1X_1 + \ldots + c_nX_n$
$$\mu_y = c_1\mu_1 + \ldots + c_n\mu_n$$
$$\sigma_y^2 = c_1^2\sigma_1^2 + \ldots + c_n^2\sigma_n^2$$

③ $X_1, \ldots, X_n$ Ind ~ ~
each with mean μ and var σ^2

$$y = \frac{X_1 + \ldots + X_n}{n}$$

Then $\mu_y = \mu$ $\sigma_y^2 = \frac{\sigma^2}{n}$

Central limit Theorem

$n > 30 \Rightarrow$ normal approximation
$n < 30 \Rightarrow$ not severly non-normal distribution
$n = 4, 5 \Rightarrow$ continuous distribution

Scanned with CamScanner

Estimation of Parameters:

$\hat{\theta}$ is an estimator of θ

- It is unbiased if $E(\hat{\theta}) = \theta$
- ~ ~ biased if $E(\hat{\theta}) - \theta = B$

- Maximum likelihood Technique:-

first find $L(\theta)$

$$L(\theta) = f(x_1, x_2 \dots ; \theta) = f(x_1) f(x_2) \dots f(\theta)$$

Then put $\frac{d\, L(\theta)}{d\theta} = 0$ or $\frac{d \ln\{L(\theta)\}}{d\theta} = 0$

Finding Internal Estimators for the Mean & Variance

C.I for Mean with variance Known :-

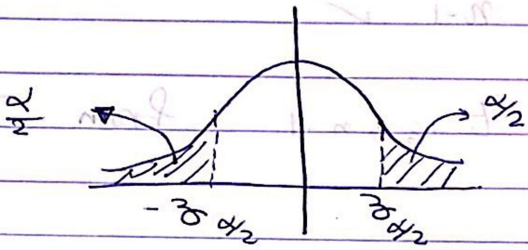
$$P\left(\hat{\mu}_x - \frac{\sigma_x}{\sqrt{n}}\, z_{\alpha/2} \le \mu_x \le \hat{\mu}_x + \frac{\sigma_x}{\sqrt{n}}\, z_{\alpha/2}\right) = 1-\alpha$$

$\hat{\mu}_x$: sample Mean $= \frac{1}{n}\sum x_i$

σ_x^2 : true variance $\Rightarrow$ Given $(\sigma_x = \sqrt{\sigma_x^2})$

$z_{\alpha/2}$: Find it from Guassian .Dist

$P(X \ge z_{\alpha/2}) = \alpha/2$



~~from~~ $P(X \le z_{\alpha/2}) = 1 - \frac{\alpha}{2}$

$\Phi(z_{\alpha/2}) = 1 - \frac{\alpha}{2}$

- When n is large the interval is smaller (more precised)

- error Between μ_x & μ_x' is :- $\mu_x - \hat{\mu}_x$

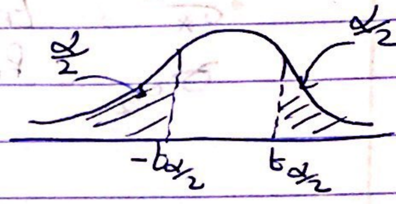
$$P\left(-\frac{\sigma_x}{\sqrt{n}}\, z_{\alpha/2} \le \mu_x - \hat{\mu}_x \le \frac{\sigma_x}{\sqrt{n}}\, z_{\alpha/2}\right) = 1-\alpha$$

CI on the Mean with Variance unknown :-

$$P\left(\hat{\mu}_x - \frac{\hat{\sigma}_x}{\sqrt{n}} t_{\frac{\alpha}{2},n-1} \leq \mu_x \leq \hat{\mu}_x - \frac{\hat{\sigma}_x}{\sqrt{n}} t_{\frac{\alpha}{2},n-1}\right) = 1-\alpha$$

$$\hat{\mu}_x = \frac{1}{n} \sum x_i$$

$$\hat{\sigma}_x = \frac{1}{n-1} \sum (x_i - \hat{\mu}_x)^2$$



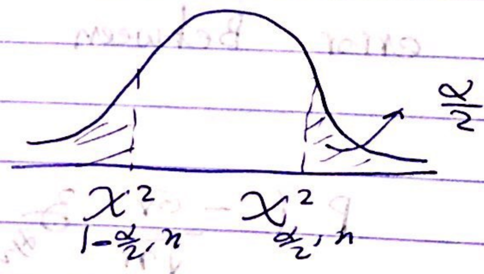
$n-1$ ✓

$t_{\frac{\alpha}{2};n-1}$ from Table ②

CI on the σ_x^2 with Mean Known :-

$$P\left(\frac{n\hat{\sigma}_x^2}{\chi^2_{\frac{\alpha}{2},n}} \leq \sigma_x^2 \leq \frac{n\hat{\sigma}_x^2}{\chi^2_{1-\frac{\alpha}{2},n}}\right) = 1-\alpha .$$

$\chi^2_{\frac{\alpha}{2},n}$ from table 3.



CT on the σ_x^2 with Mean unknown

$$P\left(\frac{(n-1)\hat{\sigma}_x^2}{\chi^2_{\alpha/2,\,n-1}} \leq \sigma_x^2 \leq \frac{(n-1)(\hat{\sigma}_x)^2}{\chi^2_{1-\alpha/2,\,n-1}}\right) = 1-\alpha$$

$\chi^2_{\alpha/2,\,n-1}$ from table 3