Approximations of Distributions :-

• Binomial $\longrightarrow$ Gaussien (De-moiver Laplac

$\mu_x = np$
$\sigma_x^2 = np(1-p)$

• To find sol in Approximation :

$$p(X \leq x) = F_x(x) = \Phi\left(\frac{X - \mu_x}{\sqrt{\sigma_x^2}}\right)$$

$$= \Phi\left(\frac{x - np}{\sqrt{np(1-p)}}\right)$$

Better results in $np > 5$

• Poisson $\longrightarrow$ Gaussien

$$e^{-b}\frac{b^x}{x!}$$

$\mu_x = \sigma_x^2 = b = \lambda T$

• To find sol in Approximation :-

$$p(X \leq x) = F_x(x) = \Phi\left(\frac{X - b}{\sqrt{b^2}}\right)$$

Better results in $b > 5$

Transformation:

• **Discrete:**

when $y = g(x)$: and X is a r.v

• To find pmf of y:-
find values of y by applying $g(x)$

Then find probability of each value
either $P(X=x) = P(Y=y)$
or sometimes Y can have the value repeated so the p is the Sum.

• **Continuous:-**

$Y = g(X)$ $\quad$ Y, X are Both r.v

$$f_y(y) = \frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{X=g(y)}$$

Note 1- If f_X is uniform & linear Then f_Y is uniform

2- when $X = g(y)$ has more than one value You add them :-

$$f_Y(y) = \frac{f_X(x)}{|\;|}\Bigg|_{X=g'(y)_1} + \frac{f_X(x)}{|\;|}\Bigg|_{X=g'(y)_2} + \cdots$$

3- Tran. of a Gaussien r.v is also Gaussien
Y has a $\mu_y = a\mu_x + b$ and $\sigma_y^2 = a^2\sigma_x^2$
where $Y = aX + b$

Two or more Random variables

- If **Discrete**

$$\sum_x \sum_y P_{xy} = 1$$

If x and y are independant Then

$\Rightarrow P(X \le x, Y \le y) = P(X \le x) P(Y \le y)$

To find PMF of x:

$$P(X=x) = \begin{cases} P(X=x_1, y=y), & X = x_1 \\ + \\ P(X=x_2, y=y), & X = x_2 \\ + \cdots\cdots, & X = x_3 \end{cases}$$

جمع كل الاحتمالات لـ X_1 و مختلف قيم y

$\Rightarrow$ To make sure X, y are independant We try each pair

$$P(X=x, y=y) = P(X=x) P(Y=y)$$

Note: 1- $F_{X,Y}(-\infty, -\infty) = 0$

$F_{X,Y}(\infty, \infty) = 1$

2- when You say : $P(X \cap Y)$ It $= P(X=x, y=y)$

• If **Continous**

$$F_{xy}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{xy}(x,y)\, dy\, dx$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x,y)\, dx\, dx = 1$$

$$P(x_1 < X \leq x_2,\ y_1 \leq Y \leq y_2) = \int_{x_1}^{x_2}\int_{y_1}^{y_2} f_{xy}\, dy\, dx$$

▸ Marginal function :-

$$\left.\begin{aligned} f_x(x) &= \int_{-\infty}^{\infty} f_{xy}(x,y)\, dy \\ f_y(y) &= \int_{-\infty}^{\infty} f_{xy}(x,y)\, dx \end{aligned}\right\} \text{Marginal Pdf}$$

▸ Independance:

$f_{xy}(x,y) = f_x(x)\, f_y(y)$ if x, y are ind

▸ Conditional probability Density function:-

$$f_{y/x} = \frac{f_{xy}(x,y)}{f_x(x)}$$

Scanned with CamScanner

Addition of Mean and variance :-

if $Y = aX + b$

Then:

$$E\{Y\} = a\,E\{X\} + b$$

$$\mu_Y = a\,\mu_X + b$$

$$\sigma_Y^2 = a^2 \sigma_X^2$$

Now:-

$$E\{g(x,y)\} = \iint_{-\infty}^{\infty} g(x,y)\, f_{xy}(x,y)\, dx\, dy \quad \text{Cont}$$

$$= \sum_x \sum_y g(x,y)\, P(X=x, Y=y) \quad \text{Dis}$$

$$E(X_1 + X_2 + \dots) = E(X_1) + E(X_2) + \dots$$

$$E(X_1 X_2 \dots) = E(X_1)\, E(X_2) \dots \quad \text{if } X_1, \dots, X_n \text{ are Ind}$$

Correlation Coefficient :-

Covariance (μ_{xy})

$$\rho_{xy} = \frac{E\{(X-\mu_X)(Y-\mu_Y)\}}{\sigma_X\,\sigma_Y}$$

$$\mu_{xx} = \sigma_X^2$$

$$\mu_{yy} = \sigma_y^2$$

ρ_{XY}

$\rho_{XY} = 0$ — X, y are uncorrelated

$\rho_{XY} = \pm 1$ — X, y are fully collerated

Note : • if X, y are ind $\rightarrow$ uncorrelated $\rho = 0$

• The inverse is not necesserily true
But if X, y are Gaussien $\Rightarrow \rho_{XY} = 0$
Then X, Y are independant

Theorem :- $\sigma_Y^2 = a_1^2 \sigma_{X_1}^2 + a_2^2 \sigma_{X_2}^2$

where $Y = a_1 X_1 + a_2 X_2$

$\rightarrow$ if X_1, X_2 are S.I

If not $\sigma_Y^2 = a_1^2 \sigma_{X_1}^2 + a_2^2 \sigma_{X_2}^2 + 2 a_1 a_2 \sigma_{X_1} \sigma_{X_2} \rho_{X_1 X_2}$

Functions of Random Variables :-

$z = g(X, Y)$ $\qquad$ Z : R.V
$\qquad\qquad\qquad\qquad$ g : function

$f_Z(z) = ?$

- **Discrete :**

$$P(Z = z) = \sum\sum P(X = x_i, \ Y = y_i)$$

- **Continuous :**

$$F_Z(z) = P(Z \le z) = \iint f_{xy}(x, y)\, dx\, dy$$

and $f_Z = \dfrac{dF_Z}{dz}$

- **Special case:**

$Z = X + Y$ $\qquad$ X, Y are S.I

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x)\, f_Y(z - x)\, dx$$