Proofs chapter 1

$P(A^c) = 1 - P(A)$

$S = A \cup A^c$

$P(S) = P(A) + P(A^c)$ $\quad A, A^c$ are disjoint

$1 = P(A) + P(A^c)$

$P(A^c) = 1 - P(A)$

$P(\phi) = 0$

$S = S \cup S^c$

$S = S \cup \emptyset \quad \Rightarrow \quad S^c = \emptyset$

$P(S) = P(S) + P(\emptyset)$

So $P(\emptyset) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ where A, B are not disjoint

$A \cap B \neq \phi$

from Fig 1:

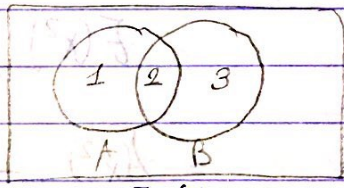


Fig (1)

$A \cup B = 1 \cup 2 \cup 3$

$P(A) = P(1) + P(2)$

$P(B) = P(2) + P(3)$

$P(A \cup B) = P(1 \cdot 2 \cdot 3 + P(2) - P(2)$

$= \{P(1) + P(2)\} + \{P(3) + P(2)\} - P(2)$

$= P(A) + P(B) - P(A \cap B)$

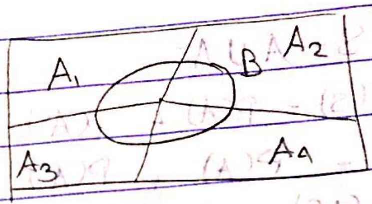
Theorem of Total Probability:-

$$P(B) = P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + \text{---}$$

$$B = \{A_1 \cap B\} \cup \{A_2 \cap B\} \cup \{A_3 \cap B\} \cup \{A_4 \cap B\}$$

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \dots$$

But: $P(A \cap B) = P(A/B)\,P(B) + P(B/A)\,P(A)$



$$P(B) = P(B/A_1)\,P(A_1) + \text{----}$$

Chapter 2

$$\sigma_x^2 = E(X^2) - \mu_x^2$$

$$\sigma_x^2 = E\{(X - \mu_x)^2\} = \int_{-\infty}^{\infty} (x^2 - 2x\mu_x + \mu_x^2) f_x(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^2 f_x(x) - \int_{-\infty}^{\infty} 2x\mu_x f_x(x)\,dx + \int_{-\infty}^{\infty} \mu_x^2 f_x(x)\,dx$$

$$= E(X^2) - 2\mu_x^2 + \mu_x^2$$

$$= E(x^2) - \mu_x^2 = E(x^2) - (E(x))^2$$

let $Y = aX + b$

$\mu_y = a\mu_x + b \qquad \sigma_y^2 = a^2 \sigma_x^2$

$\mu_y = E\{a\mu_x + b\}$

$$= \int_{-\infty}^{\infty} (a\mu_x + b) f_x(x)\,dx$$

$$= a\int_{-\infty}^{\infty} \mu_x f_x + \int_{-\infty}^{\infty} b f_x\,dx = a\mu_x + b$$

$$\sigma_y^2 = a^2\sigma_x^2$$

$$\sigma_{y^2} = E\{[y-\mu_x]^2\}$$

$$= E\{[(ax+b)-(a\mu_x+b)]^2\}$$

$$= E\{[a(x-\mu_x)]^2\}$$

$$= E\{a^2(x-\mu_x)^2\}$$

$$= a^2 E\{(x-\mu_x)^2\} = a^2\sigma_x^2$$

If X is a Binomial r.v with n, p then

$$\mu_x = E\{x\} = np.$$

$$\sigma_x^2 = Var\{x\} = np(1-p)$$

$\mu_x = np$:

$$\sum_{x=0}^{n} x\, p(X=x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x}$$

let $u = x-1$

$$= \sum_{u=0}^{n-1} \frac{n(n-1)!}{u!\,(n-1-u)!} p^{u+1} (1-p)^{n-1-u}$$

let $m = n-1$

$$= np \underbrace{\sum_{u=0}^{m} \frac{m!}{u!\,(m-u)!} p^{u} (1-p)^{m-u}}_{\text{summation} = 1}$$

$$\mu_x = np$$

$$E\{X(X-1)\} = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^{n} \frac{n!}{(x-2)!\,(n-x)!} p^x (1-p)^{n-x}$$

$u = x-2 \qquad m = n-2$

$$= \sum_{u=0}^{m} \frac{n(n-1)\,m}{u!\,(n-x)!} p^{u+2} (1-p)^{n-2-u}$$

$$= n(n-1)p^2 \left(\sum_{u=0}^{m} \frac{m}{u!} \cdots\right) \to ①$$

$$E\{X(X-1)\} = n(n-1)p^2$$

$$E(X^2) = E(X) + E(X^2 - X)$$

$$\sigma_x^2 = E(X^2) - (\mu_x)^2 = E(X) + n(n-1)p^2 - (np)^2$$

$$= np + n(n-1)p^2 - (np)^2$$

$$= np(1 + (n-1)p - np)$$

$$= np(1 + np - p - np)$$

$$\sigma_x^2 = np(1-p)$$

In Geiometric:

$$\mu_x = \frac{1}{p}$$

$$\left(1 + 2(1-p) + \cdots\right) = \left(\frac{1}{1-u}\right)$$

$$1 + 2u + 3u^2 = \left(\frac{1}{1-u^2}\right)$$

$$\mu_x = E\{x\} = \sum_{x=1}^{\infty} x p (1-p)^{x-1}$$

$$= p\left\{1 + 2(1-p) + 3(1-p)^2 + \cdots\right\}$$

$$= p\frac{1}{(1-u)^2} = p\frac{1}{(1-(1-p))^2} = \frac{1}{p}$$

$$E(X(X-1)) = \sum X(X-1)(p)(1-p)^{x-1}$$

$$= p\left\{2(1-p) + (3)(2)(1-p)^2 + \cdots\right\}$$

$$= p(1-p)\left\{2 + 6(1-p) + \cdots\right\}$$

$$2 + (3)(2)u + \cdots = \frac{2}{(1-u)^3}$$

$$= p(1-p)\frac{2}{p^3} = \frac{2(1-p)}{p^2}$$

$$\sigma_x^2 = E(X^2) - (\mu_x)^2$$

$$= \frac{1}{p} + \frac{2(1-p)}{p^2} - \frac{1}{p^2} = \frac{(1-p)}{p^2}$$

In Poissons

$\mu_x = b$

$\sigma_x^2 = b$

$\rightarrow \mu_x = b$:-

$$E(x) = \sum_{x=0}^{\infty} x e^{-b}\frac{b^x}{x!} = \sum_{x=1}^{\infty} e^{-b}\frac{b^x}{(x-1)}$$

let $u = x-1$

$$E(x) = \sum_{u=0}^{\infty} e^{-b}\frac{b^{u+1}}{u!} = b\underbrace{\sum_{u=0} e^{-b}\frac{b^u}{u!}}_{1} = b$$

$\rightarrow \sigma_x^2 = b$

$$E(X(X-1)) = \sum_{x=0}^{\infty} X(X-1)e^{-b}\frac{b^x}{X!} = \sum_{X=2}^{\infty} \frac{e^{-b}b^x}{(X-2)!}$$

let $u = X-2$

$$= \sum_{u=0}^{\infty}\frac{e^{-b}b^{u+2}}{u!} = b^2\underbrace{\sum_{u=0}^{\infty}\frac{e^{-b}b^u}{u!}}_{1} = b^2$$

$$\sigma_x^2 = -(E(x))^2 + E(x^2)$$

$$= -b^2 + \left(E(x(x-1)) + E(x)\right)$$

$$= -\cancel{b^2} + \cancel{b^2} + b = b$$

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